Notes on International Macroeconomics and Finance

Qin Lei

Georgetown University

Economics Department

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CONTENTS

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Subject: Basic Relationships and Balance of Payments Accounting

References: Bosworth, B. (1993) "Introduction" and "Alternative Measures of the Current Account" in *Saving and Investment in a Global Economy*, Washington DC: The Brookings Institution, pp. 1-37.

Caves, R., J, Frankel and R. Jones (1993) "The Balance of Payments Accounts," in *World Trade and payments: an introduction*, 6th edition, Chapter 16, pp. 305-321.

Dornbusch, R. (1980) *Open Economy Macroeconomics*, Chapter 2, New York: Basic Books.

There are three basic accounting identities:

(1) BOP Accounts $CA + CAP = 0$, $CA + KA = \Delta R$

(2) NIPA accounts $Y = C + I + G + (X - M)$

(3) Monetary accounts $DC + R = M$, $\Delta DC + \Delta R = \Delta M$

Note that there are certain links among these three accounting identities, say the change of foreign reserve ΔR between (1) and (3), and the net export amount $X - M$ between (1) and (2) through the rough approximation $CA \approx X - M$.

Let's consider the monetary accounts at first since it is the simplest one. From the viewpoint of the central bank for a country, the assets side of the T-account consists of both domestic credits DC and domestic currency value of foreign reserves R , and the liability side of the T-account is simply the money supply M . And thus we have $DC + R = M$, which also implies $\Delta DC + \Delta R = \Delta M$.

Before we consider the National Income and Products Accounts (NIPA), let's at first distinguish two confusing concepts of income Y , GNP vs. GDP. GNP is the value of final output produced by nationals regardless where they live, but GDP is just the value of final output produced within domestic borders. Although NIPA is very easy to understand, it is helpful to keep in mind that the net export $X - M$ also includes the receipts from net factor incomes.

Current account CA is defined as the record of all across border transactions of goods and services. If we adopt the approximation $CA \approx X - M$, then the NIPA directly implies that one way of looking at CA is $CA = Y - (C + I + G)$, i.e.,

[1] $CA = income - absorption$.

If we split the domestic absorption part so that $CA = Y - (C + G^c) - I = S - I$, we have

[2] $CA = saving - investment$.

Here we put a superscript ^{c} for government spending to remind ourselves that it is simply government consumption. Furthermore, if we split the saving into private part and government part, we can have various ways to express the current account:

[2.a.] $CA = (Y - C - T) + (T - G^c - I^g) - I^p = S^p - I^p + govt.budgt.surplus$ or

[2.b.] $CA = (S^p + S^g) - I$.

To understand all these different ways of looking at current account, it is important to keep in mind that essentially current account is the net saving or net wealth accumulation of the whole country.

In terms of Balance of Payment (BOP) accounts, we need to understand the definitional categories for current account CA and capital account CAP. Current account includes four major groups, merchandise that is mainly net export $X - M$, services goods, net investment income like the investment interest payment, and unilateral transfers that include both government grants and private remittances. Among these groups, we should pay special attention to the third one, the net investment income. The bookkeeping of current account is very simple and intuitive.

There are two major categories in capital account CAP, the official transactions related to the official monetary accounts and the "private" transactions KA. They are called "private" as opposed to the official ones that are related to the monetary accounts, but they also include the government's transaction not including in the monetary accounts. Basically, the private capital account KA includes both direct investment and portfolio investments. The official reserve transactions include both changes in foreign central banks' holding of domestic assets and changes in domestic central banks' holding of foreign assets, and the typical assets are gold, IMF credits and SDRs, and foreign exchange reserves.

The bookkeeping of the capital accounts is not so confusing if we refer to the following table.

(-)	Debit	Domestic residents' holdings of foreign assets $\uparrow$; Foreign agents' holdings of domestic assets $\downarrow$.
(+)	Credit	Domestic residents' holdings of foreign assets $\downarrow$; Foreign agents' holdings of domestic assets $\uparrow$.

When we are dealing with the bookkeeping, remember the following rules: Firstly, the inflow (outflow) of goods services and domestic holdings of foreign assets are debits (credits). (Note that unilateral transfers are exceptional in that the receipt of transfers is credit.) Secondly, the increase in foreign assets holding for the domestic agents is equivalent to the decrease of domestic assets holding for the foreign agents. Thirdly, relate the capital inflow (outflow) to decrease (increase) of domestic holdings of foreign assets. In one word, think anything related to capital account in terms of the change of net foreign assets holding.

Here are a few examples of the bookkeeping in the BOP accounts.
① An investment abroad would result in the increase in domestic holding of foreign assets, which is regarded as inflow of foreign assets, and thus it counts as debit on the private capital account KA.

② The purchase of machine tools bolted down to a factory floor in Scotland is similar to the purchase of Scottish machine tools imported into the United States; in the former case the debit appears on the capital account and in the latter case it appears on the merchandise trade account.

③ American acquisition of a short-term asset in another country should count as a debit on the short-term portfolio capital account since the domestic holdings of foreign assets have been increased and thus regarded as an inflow.

④ If American citizens resell to a foreign resident a bond originally issued by a foreign government, or any other foreign asset that they acquired in the past, that too counts as a credit. The reasoning is simple as follows: the domestic holdings of foreign assets have been decreased and thus regard as an outflow. Similarly, if an American buys back a US treasury bill from a foreign resident who acquired it in the past, it counts as a debit in that the decline of foreign holdings of domestic assets is equivalent to the rise of domestic holdings of foreign assets, and thus it is an inflow of assets.

⑤ When the domestic central bank buys foreign currency or gold its purchases count as a debit, just as when a private investor does so, but it appears on the official reserves transaction account rather than the private capital account.

⑥ Suppose a Japanese company buys an office building in LA and pays by check. The US BOP registers a credit under direct investment (the foreign holdings of US assets is increased) and a debit under banking flows (an American company has increased its holding of short-term claims on foreigners - it has the Japanese check).

⑦ Suppose an American firm buys a Canadian bond and pays by check, the US BOP shows a debit on portfolio capital (the firm has increased its holdings of foreign securities) and a credit under banking flows (a foreign firm has increased its holdings of short-term claims on Americans).

⑧ If an American buys a three-month Certificate of Deposit in the United Kingdom and pays by check, both the credit and the debit appear under banking flows (two short-term assets have been exchanged).

For most purposes in economics, the only concern is net flows, or total credits minus total debits. Because of the double-entry bookkeeping, we have $CA + KA - \Delta R = 0$ or $CA + KA = \Delta R$. For the sake of convenience, we let positive ΔR stand for the increase in net domestic holdings of foreign assets, and thus it counts as a debit, which is why we put a minus sign in front of ΔR.

From the viewpoint of components of current account and capital account, we have the following two ways of looking at current account:

[3] $CA = merchandise(X - M) + investment\ \ income + unilateral\, transfers$, and

[4] $CA = -CAP (= \Delta R - KA) = \Delta NFA$.

01/19/1999

Subject: Dependent Economy Model (I)
Reference: Dornbusch, Rudiger (1980) *Open Economics Macroeconomics*, Chapter 6, New York: Basic Books
Obstfeld, Maurice and Kenneth Rogoff (1996) *Foundations of International Macroeconomics*, Chapter 4, pp. 199-216

Before we move on to the Dependent Economy Model, we need to define exchange rate formally. **Nominal exchange rate** S (which comes from the spot rate) is defined to be the number of units of home country currency per unit of foreign currency. Suppose the home country is Mexico, and the foreign country is the US, then $S = peso/\$$. The relation behind is as follows: Depreciation of home currency $\Leftrightarrow S\uparrow\Leftrightarrow$ home currency is less valuable. Here we are talking about bilateral exchange rate. Sometimes we need to consider multi-lateral exchange rate or effective exchange rate.

In terms of **real exchange rate** q, we basically have three alternatives listed as follows.

(1) The approach of basket of goods: $q = SP^* \big/ P = \frac{1}{P} \Big/ \frac{1}{SP^*}$

We know that $1/P$ is the number of baskets of domestic goods for one unit of domestic currency, and that $1/(SP^*)$ is the number of baskets of foreign goods for one unit of domestic currency. The real exchange rate is just the number of baskets of domestic goods per basket of foreign goods. The relation behind is as follows: Domestic real depreciation $\Leftrightarrow q\uparrow\Leftarrow P\downarrow$.

(2) The approach of competitiveness (value-added): $q = SW^* \big/ W = \frac{1}{W} \Big/ \frac{1}{SW^*}$

Here $1/W$ is the unit of domestic labor required to produce one unit of goods, and $1/(SW^*)$ is the unit of foreign labor required. So basically we are comparing the labor costs across countries to get the real exchange rate. The relation behind is as follows: Domestic real depreciation $\Leftrightarrow q\uparrow\Leftarrow W\downarrow$.

(3) The approach of tradable vs. nontradable: $q = P_T / P_N$

This version of definition is very popular in the model without money. The relation behind is as follows: Domestic real depreciation $\Leftrightarrow q\uparrow\Leftarrow P_N\downarrow$. In a model with this version of real exchange rate, if the home country experiences a CA deficit, that means it imports from abroad, by looking at the CA in the way of $CA = income - absorption$. Recall that in trade theory the terms of trade TOT is defined as $TOT \equiv P_X / P_M$. In this version of real exchange rate we can construct a tradable goods that is a composite of both import and export goods, whereas in the first version of exchange rate P and P^* can include both P_X and P_M.

Now we are ready for the Dependent Economy Model, which is also called Salter-Swan Model. Suppose we are considering a small open economy. By small we mean the country takes the key world prices as given, and here the terms of trade in the country considered is also given. This country produces two composite goods, tradable and nontradable, and the nontradable is taken as the *numeraire* goods. We assume there is a full employment at the level of $\overline{L}$ in this economy, while the sector-specific capital stocks are $\overline{K}_T$ and $\overline{K}_N$, respectively.

There are some obvious shortcomings in this model: this is not an intertemporal model; it is not an optimizing model; and there is no international capital mobility here. Despite these shortcomings, we have quite a few nice results regarding the production and labor market, listed as follows.

[1] $Y_T = F(L_T;\overline{K}_T) = Y_T(P_T/P_N;...) = Y_T(+;...)$

[2] $Y_N = G(L_N;\overline{K}_N) = Y_N(P_T/P_N;...) = Y_N(-;...)$

[3] $Y \equiv Y_N + (P_T/P_N)Y_T = Y(P_T/P_N;...) = Y(+;...)$

In these results, the endogenous variables are listed in front of the semi-colon whereas the exogenous ones are behind it. The plus and minus sign in the last equation indicates the comparative static effects for corresponding variables.

There are two alternative ways to see why we can write the production as a function of the real exchange rate, i.e., the tradable vs. nontradable prices ratio. The first one is a bit intuitive. Consider the PPF curve in Figure 1. Given the price ratio, we can obviously determine the specific production by finding the tangent point of the PPF curve with the budget constraint that has the slope of prices ratio. Given a nice curvature of the PPF curve, the production is uniquely determined by the tradable vs. nontradable price ratio, and thus the second equal sign above holds. Moreover, the intersection point of the budget constraint with the vertical axis measures the value of total production in terms of non-tradable goods, and this is what the first "assign" sign in result [3] stands for. Finally, to get the comparative static effects, we need to change the slope of the budget constraint, say increase the relative price. It is easy to see that the production point should move downward, and thus $Y_T \uparrow, Y_N \downarrow$ and $Y \uparrow$.

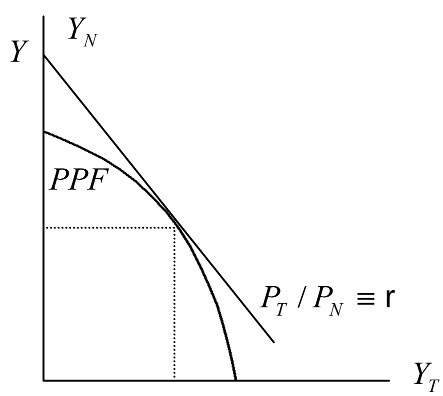

Figure 1

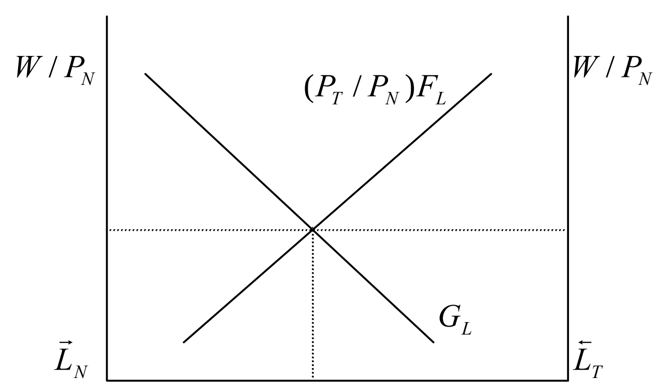

Figure 2

The second way of seeing why we can write the production as a function of the relative price is to work through the labor market equilibrium. The equilibrium conditions for tradable and nontradable sectors are $W/P_T = W/P_N \cdot P_N/P_T = F_L(L_T;\overline{K}_T)$ and $W/P_N = G_L(L_N;\overline{K}_N)$. From these two equilibrium conditions we can easily find the derived labor demand for each sector as follows, $L_T^d = L_T^d(W/P_N, P_T/P_N;\overline{K}_T) = L_T^d(-,+;+)$ and $L_N^d = L_N^d(W/P_N;\overline{K}_N) = L_N^d(-;+)$.

The individual comparative static effects are derived in the following way:

$$W/P_N \uparrow \Rightarrow L_T \downarrow;\ P_T/P_N \uparrow \Rightarrow W/P_T \downarrow \Rightarrow L_T \uparrow;\ \overline{K}_T \uparrow \Rightarrow F_L \uparrow \Rightarrow L_T \uparrow;$$
$$W/P_N \uparrow \Rightarrow W/P_T \uparrow \Rightarrow L_N \downarrow;\ \overline{K}_N \uparrow \Rightarrow F_L \uparrow \Rightarrow L_N \uparrow.$$

From the full employment condition, we have

$$[4]\ \overline{L} = L_T^d(W/P_N, P_T/P_N; \overline{K}_T) + L_N^d(W/P_N; \overline{K}_N) = L_T^d(-,+;+) + L_N^d(-;+).$$

Therefore, given the full employment condition, once the relative price P_T/P_N is known, the wage rate in terms of *numeraire* W/P_N can be derived, and thus both Y_T and Y_N are determined, which is what we claimed above. In the meanwhile, the labor allocation across sectors is also determined by the relative price. The relationship above, as well as the comparative static effects can be well represented in Figure 2.

It is intuitive that the wage in terms of tradable W/P_T is also kind of like real exchange rate, and it would be nice to develop a relationship between this and the popular version of real exchange rate, i.e., the relative price P_T/P_N. We can rewrite the full employment condition as the following

$$\overline{L} = L_T^d(W/P_T; \overline{K}_T) + L_N^d(W/P_T, P_T/P_N; \overline{K}_N) = L_T^d(-;+) + L_N^d(-,-;+).$$

01/21/1999

Subject: Dependent Economy Model (II)
Reference: Dornbusch, Rudiger (1980) *Open Economics Macroeconomics*, Chapter 6, New York: Basic Books
Obstfeld, Maurice and Kenneth Rogoff (1996) *Foundations of International Macroeconomics*, Chapter 4, pp. 199-216

As a review of last class, we have the following results.

[1] $Y_T = F(L_T;\overline{K}_T) = Y_T(P_T/P_N;...) = Y_T(+;...)$

[2] $Y_N = G(L_N;\overline{K}_N) = Y_N(P_T/P_N;...) = Y_N(-;...)$

[3] $Y \equiv Y_N + (P_T/P_N)Y_T = Y(P_T/P_N;...) = Y(+;...)$

[4] $\overline{L} = L_T^d(W/P_T;\overline{K}_T) + L_N^d(W/P_T, P_T/P_N;\overline{K}_N) = L_T^d(-;+) + L_N^d(-,-;+)$

Equation [1] and [2] described the two production sectors. The total value of production in terms of the *numeraire* good, nontradable, is represented by equation [3], which is called YY schedule and is upward sloping. The full employment condition on the labor market implies equation [4].

We can also rewrite the full employment condition as $\overline{L} = L^d(W/P_T, P_T/P_N) = L^d(-,-)$. From this representation, we know there is a negative relation between the wage in terms of tradable W/P_T and the real exchange rate P_T/P_N. This is graphically represented by the downward sloping LL schedule in Figure 3.

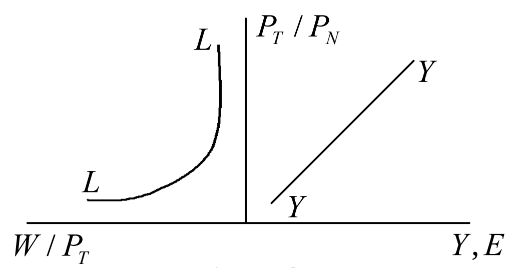

Figure 3

After having considered the supply side of the economy, it is time to consider the demand side, which can be represented by the following equations.

[5] $E = D_N + (P_T/P_N)D_T$

[6] $Y_N(P_T/P_N) = D_N(P_T/P_N, E)$

[7] $Y_T(P_T/P_N) = D_T(P_T/P_N, E)$

It is straightforward that the total expenditure E consists of demand for nontradable D_N and this can be written in terms of *numeraire* goods as equation [5]. The market equilibrium condition for nontradable goods is equation [6], and the positive relationship between real exchange rate and demand results from the usual assumption that the substitution effect dominates the income effect. Similarly, equation [7] gives the market clear condition for the tradable goods. Note that since in this economy only tradable goods can be exported and/or imported, equation [7] is also the CA balance condition.

From equation [6], we can derive a negative relationship between the real exchange rate and the expenditure in the nontradable goods, called NN schedule. The reasoning for its downward sloping is that the expenditure has to be contracted in order to offset the excess demand for nontradable goods corresponding to the real depreciation, i.e., $P_T / P_N \uparrow \Rightarrow Y_N \downarrow$ and $D_N \uparrow \Rightarrow E \downarrow$. From [7], we find that the relationship between the real exchange rate and the expenditure in the tradable goods is positive and called TT schedule. The similar reasoning is that the expenditure has to be compensated in order to offset the excess supply of tradable goods corresponding to the real depreciation, i.e., $P_T / P_N \uparrow \Rightarrow Y_T \uparrow$ and $D_T \downarrow \Rightarrow E \uparrow$.

We have already known that the YY schedule is also upward sloping. It remains an interesting question to determine which of YY and TT schedule is steeper. Let's start from the intersection point A of YY and TT schedule. Suppose there is a real depreciation up to point B. On the supply side, we know that the production of tradable will go up and that of nontradable will go down. The total value of production in terms of *numeraire* will go up to the point C. In the meanwhile, the situation on the demand side is that demand for tradable will go down and that for nontradable will go up. The existence of excess supply of tradable calls on the expansion of expenditure. However, the same amount of expansion of expenditure as that of output, up to point C, is not enough to restore the equilibrium in the tradable goods market in that wealth effect will be split between tradable and nontradable. Therefore, we need a further expansion of expenditure, up to point D, to reach the tradable goods market equilibrium and thus a CA balance. (We are not considering the excess demand for nontradable goods here because that at point D, the nontradable goods market is not clear.) This is, the YY schedule is steeper than the TT schedule, as in Figure 4.

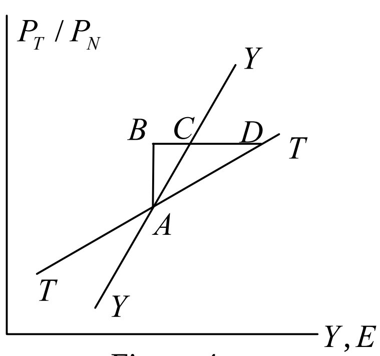

Figure 4

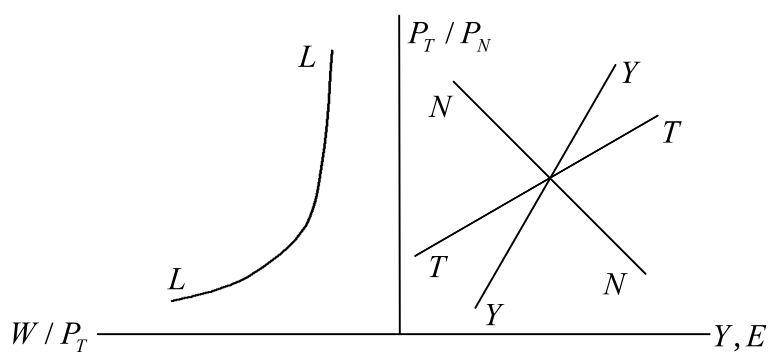

Figure 5

Notice that essentially the YY schedule is the locus of points where the income and expenditure are balanced in the economy. The TT schedule is the locus of points where CA is balanced and the tradable goods market is clear (These two conditions overlap only in this particular context of dependent economy). The NN schedule is the locus of points where the nontradable goods market is clear. Walras's law tells us that those two markets will be clear, together with the clearing in the whole economy. A mathematical demonstration of this is that $Y = E$ and $Y_N = D_N$ will ensure $Y_T = D_T$, which is easily proved by noticing that $Y = Y_N + (P_T / P_N) Y_T$ and $E = D_N + (P_T / P_N) D_T$. The graphical representation of the Walras's law is the common intersection point of the YY, TT and NN schedule, as in Figure 5.

Next, let's do a simple anatomy of the dis-equilibrium. Suppose the nontradable goods market must be clear all the time, and the economy is currently locating at point A in Figure 6. Point A tells us that the current real exchange rate is r_A, which stands for a real appreciation, and the economy's total expenditure level in terms of *numeraire* goods is E_A. From the current real exchange rate level, we find that he total actual production level in terms of *numeraire* goods is Y_A. The difference $E_A - Y_A$ is thus the CA deficit. And the fact that the economy is locating at point A, which is away from the TT schedule, is also telling us that CA is not balanced. From the graph, we also see that the wage in terms of tradable $(W/P_T)_A$ is higher than the equilibrium level. All these details exhibit the following picture: Due to the real appreciation, the production of nontradable goods goes up and that of tradable goods goes down, and the total value of the new production level goes down at level Y_A. The demand for the nontradable goods goes down and thus an expansion of the total expenditure is called for up to the level E_A to successfully clear the nontradable goods market at point A. On the side of the tradable goods, however, the excess demand for tradable goods calls on import amount of $E_A - Y_A$ in terms of *numeraire* gods to fill in the gap and thus produces the same amount of CA deficit. In the meanwhile, the real appreciation also causes the increase in wages in terms of tradable.

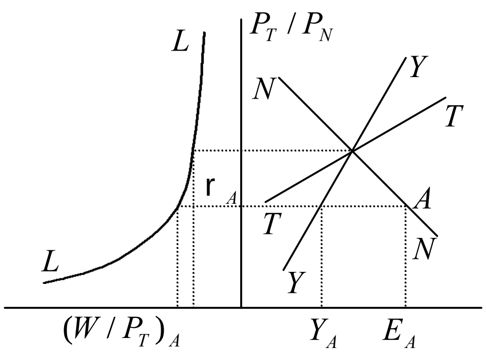

Figure 6

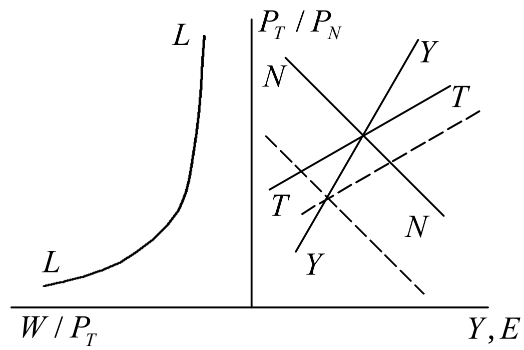

Figure 7

Lastly, let's do a simple experiment of taste switching. Suppose all of sudden, there occurs a demand shift from tradable to nontradable. We know that nothing exogenous happened to the YY schedule, and thus it remains at the original palace. At the equilibrium real exchange rate level, the demand for nontradable goods rises due to the taste switching. The excess demand for nontradable goods calls for a contraction of expenditure in order to rebuild the equilibrium on the nontradable goods market. Similarly, the excess supply of tradable goods calls for an expansion of expenditure in order to restore the market clearance. Therefore, we have a left-hand-side shift for the NN schedule and a right-hand-side shift for the TT schedule, which intersect at the YY schedule and form a new equilibrium. Figure 7 depicts responses to such a taste switching.

01/26/1999

Subject: Dependent Economy Model (III)
Reference: Dornbusch, Rudiger (1980) *Open economics Macroeconomics*, Chapter 6, New York: Basic books
Obstfeld, Maurice and Kenneth Rogoff (1996) *Foundations of International Macroeconomics*, Chapter 4, pp. 199-216

We are going to do two simple applications using the dependent economy model here. One is related to an increase of productivity in the tradable sector while the productivity in the nontradable sector remains the same as before. The other issue is the so-called "Dutch Disease."

Experiment 1: Increase of Tradable Productivity

Since there is nothing exogenous happened to the production in the nontradable sector, the NN schedule will not shift at all. The increased productivity will increase the total production value in terms of *numeraire* goods, and thus the YY schedule will shift towards the right. Since nothing happened to the demand for tradable goods, the excess supply of tradable gods will calls for the expansion of expenditure to suppress the excess supply. However, at the original relative price, the horizontal movement from point E_0 to E in Figure 8 will not be enough, or say the YY schedule and the TT schedule will not intersect at the point E_1. Since the expansion of expenditure from point E_0 to E_1 will be split between tradable and nontradable and the increase of production is purely due to the productivity growth of the tradable, this horizontal amount of expenditure expansion is not enough to fully suppress the excess supply of tradable goods. We should move the TT schedule further toward the right and thus reach the new equilibrium point E_2. Therefore, we have a real appreciation in this small economy.

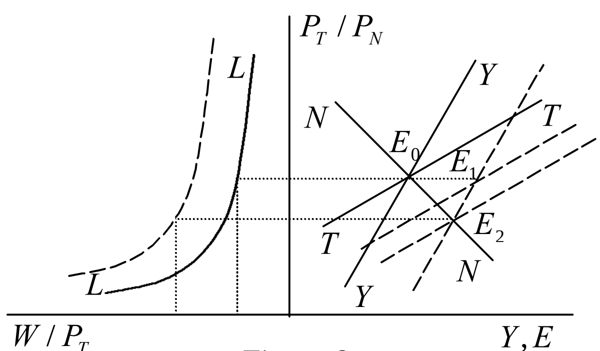

Figure 8

Next, let's consider the response in the labor market to such an increase of productivity growth in the tradable goods. At the original levels of relative price, the increased productivity induces an increased labor demand, and thus the LL schedule should move upward. Graphically in Figure 8, the new equilibrium wage level in terms of tradable, corresponding to both the real appreciation and the upward movement of the LL schedule, will rise.

We can also reason the changes in the labor market in the figure 9, which is slightly different from Figure 2 in terms of axes. Graphically, the increased productivity in the tradable goods sector forces its labor demand schedule to move upward. As a consequence, the relative productivity growth in the tradable sector will make the labor more from the nontradable sector to the tradable one.

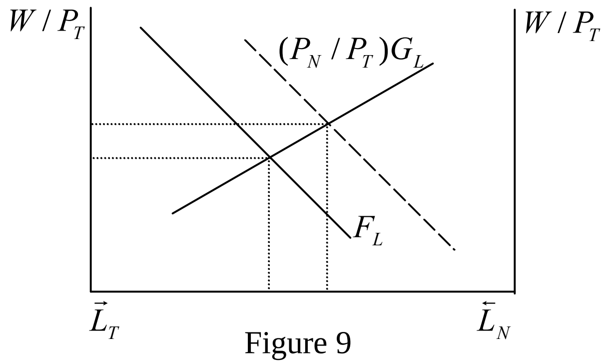

Figure 9

The fact that a relative productivity growth in the tradable sector will result in a real appreciation and thus higher domestic price level is also called Harrod-Balassa-Samuelson effect. There are a few empirical evidences support this reasoning. One is that rich countries tend to have higher price level; another on is that the price of nontradable relative to tradable tends to rise as a country develops. We also find that there is a positive correlation between the growth of nontradable prices relative to tradable prices and the growth of tradable sector productivity relative to that in the nontradable sector.

Experiment2: "Dutch Disease" Phenomenon

Suppose the small country discovered oil. Let P_o be the world price of oil, which is give, and P_T be the world price of traditional tradable goods. Then $g \equiv P_o / P_T$ is given by assumption. Suppose we don't need any factors like capital to explore the newly discovered oil for simplicity. Let O be the amount of oil discovered.

There is nothing happened in the nontradable sector, and thus the NN schedule will not change at all. In the market of tradable, we have the following change:

$$(P_T / P_N) \cdot g \cdot O + Y_T(P_T / P_N) = D_T(P_T / P_N, E).$$

Thus the YY schedule will move towards the right. Similarly, the YY schedule will also have the following change:

$$Y = (P_T / P_N) \cdot g \cdot O + Y_N + (P_T / P_N)Y_T .$$

Thus the YY schedule will also shift toward right. Once again, the same amount of horizontal movement, relative to the old equilibrium point, of the TT schedule as that of the YY schedule won't make the CA in balance and we should move further toward right until a new equilibrium point at the original NN schedule. Now we once again have a real appreciation.

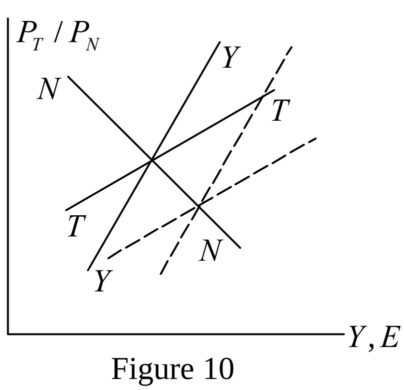

Figure 10

The so-called “Dutch Disease’ describes the phenomenon that in certain country there are some newly discovered natural resources so as to induce a real appreciation. Due to the real appreciation, the nontradable goods production becomes less competitive, and factors will shift from the nontradable sector to the tradable sector, which will shrink the nontradable production sector. One way of solving this problem will be saving those resources for the rainy day down the road and thus avoid the unfavorable factor shifting.

01/28/1999

Subject: Steady-State Model with Mobile Capital (I)
Reference: Obstfeld, Maurice and Kenneth Rogoff (1996) *Foundations of International Macroeconomics*, Chapter 4, pp. 199-216

Here in the model, we allow the mobility of capital, but not labor, and this is a more realistic assumption than in the dependent economy model. We are also going to introduce optimizing behavior into the model, and some sort of intertemporal smoothing character (Only the steady state properties across time period will be considered here). This small open economy produces two composite goods, tradable and nontradable. Note that, not the same as before, here we are using the tradable as the *numeraire* good and thus the real exchange rate is defined as $\mathrm{r} \equiv P_N / P_T$.

Suppose the real interest rate r and the price of tradable P_T is given by the rest of the world. Also note that both capital and the real interest rate are in terms of tradable goods, and the real interest rate satisfies $r = MPK_T = (P_N / P_T) MPK_N$. The wage in terms of tradable is w. Let A_T and A_N stand for the total factor productivity in two sectors, and the production function are given by $Y_T = A_T F(K_T, L_T)$ and $Y_N = A_N G(K_N, L_N)$. Assume further that there is a labor resource constraint due to the international immobility of labor, i.e., $\bar{L} = L_T + L_N$. Capital can be transported freely and used for either production or consumption. Since the production technology in this economy is CRS in both sectors, we know for sure in advance that the zero profit condition is going to hold. That is, $Y_T = wL_T + rK_T$ and $Y_N = wL_N + rK_N$. In intensive form, they are $A_T f(k_T) = w + rk_T$ and $A_N g(k_N) = w + rk_N$.

Suppose the firms are trying to maximize the present discounted value of profits. In the tradable sector, the objective function is given by

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \left[A_{T,s} F(K_{T,s}, L_{T,s}) - w_s L_{T,s} - (K_{T,s+1} - K_{T,s}) \right],$$

and the objective function in the nontradable sector is given by

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \left[\mathrm{r} \cdot A_{N,s} F(K_{N,s}, L_{N,s}) - w_s L_{N,s} - (K_{N,s+1} - K_{N,s}) \right].$$

Here we are essentially assuming that the owners of the firm also own the capital so that they don’t have to pay the interest to the owner and only the amount of net investment in each period should be deduct from the product revenue.

We have the following four first-order-condition for the firms’ optimization problem. Note that we have omitted the time subscript since we are going to consider only the steady state situation and we use the intensive production form here.

[1] $A_T f'(k_T) = r$
[2] $A_T [f(k_T) - k_T f'(k_T)] = w$
[3] $\mathrm{r} A_N g'(k_N) = r$
[4] $\mathrm{r} A_N [g(k_N) - k_N g'(k_N)] = w$

Here we have three exogenous variables, A_T, A_N, r, and four endogenous ones, $k_T, k_N, \rho \equiv P_N / P_T, w$. Before we find out the comparative static effect of these variables, it is useful to note that w is the product wage rate in terms of tradable, or more formally "tradable goods product wage". It is not the usual real wage we have in mind since the real wage would have to include both tradable and nontradable goods.

From equation [1], k_T is uniquely determined and we have

[5] $k_T = k_T(A_T, r) = k_T(+,-)$.

The reasoning for the sign behind [5] is $A_T \uparrow \Rightarrow MPK_T \downarrow \Rightarrow k_T \uparrow$ and $r \uparrow \Rightarrow MPK_T \uparrow \Rightarrow k_T \downarrow$.

Combining equation [1] and [2], we know that

[6] $w = w(A_T, k_T) = w(A_T, r) = w(+,-)$.

The reasoning for the sign behind [6] is $A_T \uparrow \Rightarrow w \uparrow; k_T \uparrow \Rightarrow w \uparrow; MPL_T \uparrow \Rightarrow w \uparrow; w \uparrow$ and $r \uparrow \Rightarrow k_T \downarrow \Rightarrow MPL_T \downarrow \Rightarrow w \downarrow$. (Note here that k_T is negatively related with MPK_T according to the law of diminishing return; but it is positively related with MPL_T according to the simple derivation: $\partial MPL / \partial k = f'(k) - [f'(k) + kf''(k)] = -kf''(k) > 0$.)

In order to mathematically determine the sign of the comparative static effects, we need to total differentiate the proper equation and find out the expression of the differential of the target variable in terms of the differential of its arguments. Suppose we would like to determine sign in [5], then we need to totally differentiate equation [1] and derive an expression like $dk_T = \alpha_A dA_T + \alpha_r dr$. In fact, if we do the algebra, we find that $\alpha_A = [-f'(k_T)]/[A_T f''(k_T)] > 0$ and $\alpha_r = 1/[A_T f''(k_T)] < 0$.

Equation [6] also shows that the wage rate is determined completely in the tradable goods sector, and only the world interest rate and the productivity in tradable sector matters. The reason why this is different from the conclusion in the Dependent Economy Model, where we have wage determined by both sectors, lies in the fact that not like in the DEM where we have sector specific capital we have mobile capital across countries and sectors in this model.

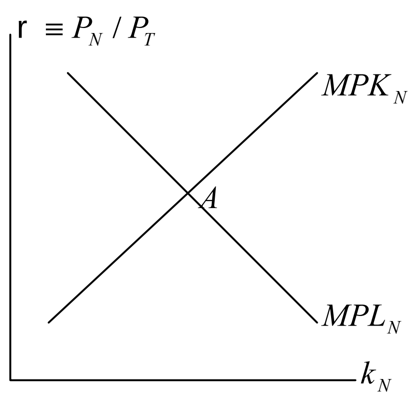

Figure 11

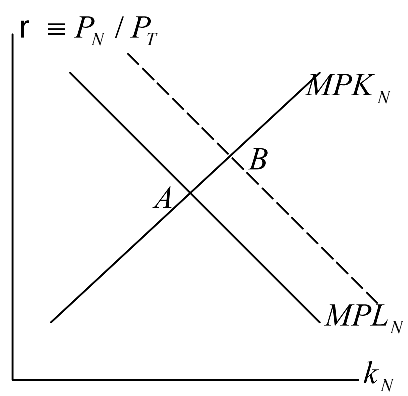

Figure 12

Now we are left with the real exchange rate ρ and k_N to be determined from equation [3] and [4]. Since the wage rate is involved in equation [4], we also know that we have to utilize the results from equation [5] and [6] as well. We want to graphically depict the relationship between the real exchange rate r and k_N. Suppose in Figure 11, there is a point A where equation [3] is satisfied. If we increase k_N horizontally from point A, we know that $MPPK_N$ will go down and thus we need to have a higher r, i.e., a real appreciation, to restore the relationship [3]. Therefore, we can derive an upward sloping MPK_N curve from [3]. Similarly, suppose equation [4] is also satisfied at point A. If we horizontally increase k_N then $MPPL_N$ will go up. We need a real depreciation to restore the relationship [4]. Therefore, we can derive a downward sloping MPL_N curve from [4].

Combining equations [1] through [4], we should be able to get the following two relations.

[7] $k_N = k_N(A_T, A_N, r)$

[8] $\mathrm{r} = \mathrm{r}(A_T, A_N, r)$

To determine the sign of the comparative static effects in these two equations, let's do the following. Suppose there is an anticipated increase in A_T. Since A_T doesn't enter equation [3], we know the MPK_N curve remains its original location. Equation [6] tells us that w would increase associated with the increase in A_T. From equation [4] we know that the increase in w will shift the MPL_N curve to the right and thus we have a new equilibrium point B in Figure 12.

Comparing point A and B, we find that the economy will experience a real appreciation associated with the growth of productivity in the tradable sector, which is exactly the same conclusion we got from the Dependent Economy Model. We also find that k_N will increase as well. The same positive impact of A_T on both k_T and k_N may drive us to ask how come both sectors become capital intensive corresponding to the growth of productivity in tradable sector? The answer is that in this economy capital is freely mobile across sectors and countries so that this country is going to import capital from outside when there is a productivity growth in the tradable sector.

It is nice to ask us the same question in the context of Edgeworth Box. We know there are some upper bound for total capital and total labor. The reason why both sectors become capital intensive corresponding to the productivity growth in the tradable sector lies in the fact that the original capital intensive sector has to shrink so that there is some capital released to the other sector. This is nicely reflected in Figure 13.

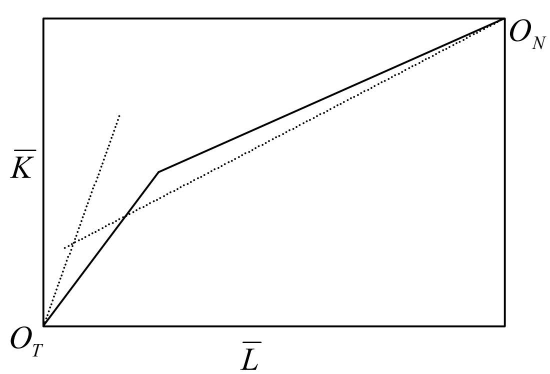

Figure 13

02/04/1999 & 02/09/1999

Subject: Steady-State Model with Mobile Capital (II)
Reference:
Obstfeld, Maurice and Kenneth Rogoff (1996) *Foundations of International Macroeconomics*, Chapter 4, pp. 199-216

In Figure 12, we have derived the positive relation between A_T and k_N, and the positive relation between A_T and $\mathrm{r} \equiv P_N / P_T$. So the equations [7] and [8] can be updated as follows.

[7] $k_N = k_N(A_T, A_N, r) = k_N(+,?,?)$

[8] $\mathrm{r} = \mathrm{r}(A_T, A_N, r) = \mathrm{r}(+,?,?)$

Next, let's consider the response of k_N and r to an increase in A_N. Equation [3] says that for given k_N, r has to decrease proportional to the increase in A_N so that the MPK_N schedule will shift downward as in Figure 14. Then equation [4] says that the change of r proportional to A_N will have k_N unchanged. Therefore, we have the downward shifting of MPL_N schedule and thus a lower equilibrium level of real exchange rate without change in k_N. Now we have the updated equations [7] and [8] as:

[7] $k_N = k_N(A_T, r) = k_N(+,?)$ and

[8] $\mathrm{r} = \mathrm{r}(A_T, A_N, r) = \mathrm{r}(+,-,?)$.

In the meantime, we also find from equation [2] that the product wage w will not change corresponding to the increase of productivity in the nontradable sector. The real wage, however, will rise because the price of nontradable goods will go down (due to the real appreciation) and thus the composite price will go down.

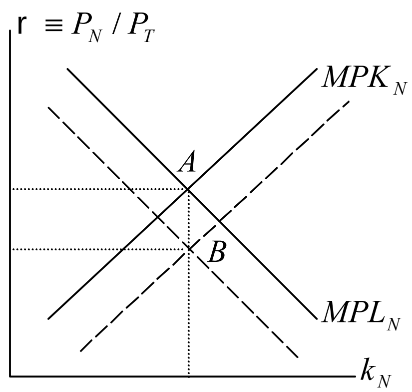

Figure 14

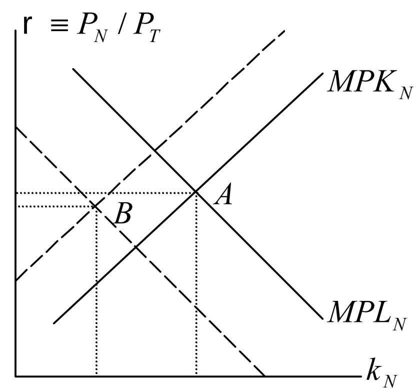

Figure 15

Lastly, we need to consider responses of k_N and r to an increase in r. Equation [3] says that for given k_N, r will increase so that the MPK_N schedule will shift upward as in Figure 15. We also know that the increase of world interest rate r will cause w to go down. Equation [4] then tells us that for given k_N, r will go down so that the MPL_N schedule will shift downward as in Figure 15. However, it is ambiguous whether r at the equilibrium will go down or up. Intuitively, the nontradable goods sector is labor-intensive so that the increase of capital rental price implies the increase in tradable goods price relative to nontradable goods price so that we would have a real depreciation. Therefore, under the assumption that the nontradable sector is labor-intensive, we have [7] and [8] as follows,

[7] $k_N = k_N(A_T, r) = k_N(+,-)$

[8] $\rho = \rho(A_T, A_N, r) = \rho(+,-,-)$

We can also rigorously prove as follows that the assumption of labor-intensive nontradable sector implies the depreciation corresponding to the increase of world interest rate.
Totally differentiating equations [1] and [2], we have

$A_T f''(k_T)dk_T = dr$ and $-A_T k_T f''(k_T)dk_T = dw$.

Those two results then imply

[9] $dw = -k_T dr$.

Similarly, totally differentiating equation [3] and [4], we have

$A_N g'(k_N)d\rho + \rho A_N g''(k_N)dk_N = dr$ and $A_N[g(k_N) - k_N g'(k_N)]d\rho - \rho A_N k_N g''(k_N)dk_N = dw$.

These two results then implies

$A_N[g(k_N) - k_N g'(k_N)]d\rho - k_N[dr - A_N g'(k_N)d\rho] = dw$, i.e.,

[10] $A_N g(k_N)d\rho = dw + k_N dr$.

If we combine equation [9] and [10], then we have

$A_N g(k_N)d\rho = (k_N - k_T)dr$.

Therefore, the assumption of labor-intensive nontradable sector, i.e., $k_N < k_T$, implies a negative relationship between the real exchange rate and the world interest rate.

Our next topic is to develop the steady-state growth path relationship between the real exchange rate and the relative productivity across sectors. The first thing we ought to note is that the CRS production technology implies the zero-profit condition holds in both sectors. In mathematical expressions, this is

[11] $A_T f(k_T) = w + r \cdot k_T$

[12] $\rho A_N g(k_N) = w + r \cdot k_N$

A normal trick to obtain steady-state growth path is to take natural logarithm on both sides of an equation and then totally differentiate it. If we do so to equation [11], we have

$$\ln(A_T) + \ln[f(k_T)] = \ln(w + rk_T)$$

$$\frac{dA_T}{A_T} + \frac{f'(k_T)dk_T}{f(k_T)} = \frac{dw + rdk_T}{A_T f(k_T)} \quad \text{(Use equation [11] here.)}$$

$$\frac{dA_T}{A_T} = \frac{w}{A_T f(k_T)}\frac{dw}{w} \quad \text{(Use equation [1] here.)}$$

If we define $\mu_{LT} \equiv w/[A_T f(k_T)]$, then we have

[13] $\hat{A}_T = \mu_{LT} \cdot \hat{w}$.

If we do the similar trick to equation [12] and define $\mu_{LN} \equiv w/[\rho A_T f(k_T)]$, then we have

[14] $\hat{A}_N + \hat{\rho} = \mu_{LN} \cdot \hat{w}$.

The combination of [13] and [14] implies the following important result:

[15] $\hat{\rho} = (\mu_{LN} / \mu_{LT})\hat{A}_T - \hat{A}_N$.

Equation [15] says that if the nontradable sector is labor intensive so that the labor income share of nontradable is higher than that of tradable, then the same growth rate of productivity in two sectors would imply the real appreciation. This is a stronger result than the one we got from DEM, which says that a higher relative productivity in the tradable sector implies a real appreciation.

Suppose in this economy the output Y is defined to be GDP, and thus the zero-profit condition implies that $Y = GDP = wL + rK$. We also assume that capital is tradable goods so that they can be invested or consumed, and the following identity holds: $K = K_T + K_N$. Let's denote by B_{t+1} the value of net foreign assets by the end of period t . We then know that $B_{t+1} > 0$ implies that the home country is a net creditor, and in this situation the interest earning will be part of CA . Let's next define the total financial wealth $Q = B + K$. Then the national income account identity implies the following: $GNP = GDP + rB = wL + rK + rB = wL + rQ$.

Suppose momentarily that the national income is equal to total consumption, then we know $CA = 0$, i.e., the trade surplus could be used to pay off the debt interest payment or the trade deficit could be paid off by the investment interest payment.

On the demand side of the economy, let's assume homothetic preferences for the sake of simplicity so that we obtain an income expansion line as shown in Figure 16. We also assume the productivity A_T and A_N in two sectors are given. At the steady state, we should have $GNP = w(r) \cdot \overline{L} + r \cdot \overline{Q} = \rho(r)\overline{C}_N + \overline{C}_T$. This implies the GNP line or the budget constraint as $\overline{C}_T = -\rho(r) \cdot \overline{C}_N + [w(r) \cdot \overline{L} + r \cdot \overline{Q}]$.

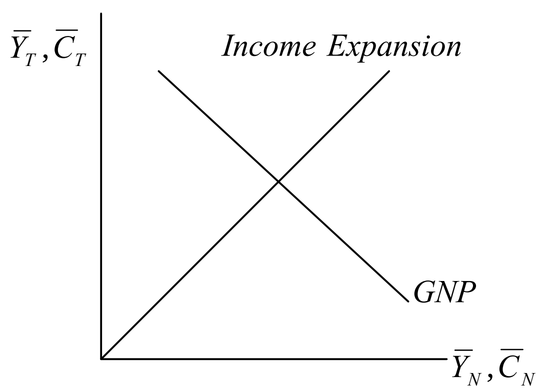

Figure 16

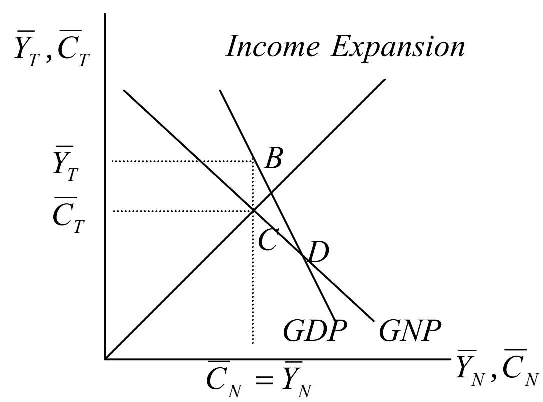

Figure 17

In the production side of the economy, the zero-profit condition in each sector implies the following:

$$\overline{Y}_T = r\overline{K}_T + w\overline{L}_T = [r\overline{k}_T(r) + w(r)]\overline{L}_T \equiv \xi_T(r)\overline{L}_T$$

$$\rho\,\overline{Y}_N = r\overline{K}_N + w\overline{L}_N = [r\overline{k}_N(r) + w(r)]\overline{L}_N \equiv \xi_N(r)\overline{L}_N$$

The combination of these two relations implies the following relationship between the production of tradable and nontradable goods in the steady state,

$$\overline{Y}_T = \xi_T(r)\overline{L} - \frac{\xi_T(r)}{\xi_N(r)}\rho(r)\overline{Y}_N .$$

We call this relationship the GDP line, which is obviously downward sloping. If we adopt the conventional assumption that the production of nontradable is labor-intensive, i.e., $\overline{k}_T > \overline{k}_N$, then by definition we have $\xi_T > \xi_N$ and thus the GDP line is steeper than the GNP line. Not like the normal PPF curve in a closed economy, this GDP line is straight. We know that in a regular closed economy, the law of diminishing return implies that the capital labor ratio would decline if we use more capital to produce. On the contrary, in this small open economy, we can import capital from outside so that the capital-labor wouldn't decline despite the increase in usage of capital.

We draw the Income Expansion line, GNP line and GDP line together in Figure 17. And we assume that those three curves do not intersect each other at the same point. The intersection point C between the Income Expansion line and the GNP line describes the consumption allocation. Since there is no way to import/export the nontradable goods in this economy, the clearance in the nontradable goods market implies that $\overline{C}_N = \overline{Y}_N$. The direct impact of this relationship is that we have determined the production level of nontradable goods, which then implies the corresponding production level of tradable goods by the GDP line. That is, the production point in this economy is point B.

The current account CA is in balance on the GNP line by construction. Because we want to produce more than demanded for tradable goods, we would import extra capital to facilitate the production. As a result, the trade surplus has to be used to pay off the interest payment to the accumulated debt. Alternatively, we can compare the intersection point D between the GNP and GDP lines with point B: at point B, we have a higher level of GDP than GNP so that we must accumulate debt.

We are not going to analyze the comparative static effects corresponding to an increase in world interest rate here, but we surely can. Suppose we compare two countries that have different financial wealth levels, due to historical reasons, instead. We should find a higher general level of GNP line for the country with higher financial wealth level. If other things are the same, the GDP lines for two countries should be the same.

02/09/1999 & 02/16/1999

Subject: Simple Intertemporal Model
Reference: Obstfeld, Maurice and Kenneth Rogoff (1996) *Foundations of International Macroeconomics*, Chapter 4, pp. 199-216

1. Simple Intertemporal Model without Capital

Before we analyze the simple two-period model, it is useful to review a few basic concepts. Suppose the world real interest rate is r. Let the current and future prices be P_1 and P_2. $1/(1+r) = P_2/P_1$ is called the intertemporal terms of trade in the sense that if we act as creditor in this intertemporal trade, then we are exporting current consumption in exchange of future consumption. If r is higher, then the future goods are relatively cheaper; as a creditor, our terms of trade is thus improving. Let B_t be the value of foreign assets stock at the end of period t. Then we have following ways of looking at the current account:

$$CA_t = (Y_t + rB_t) - (C_t + I_t + G_t) = S_t - I_t = B_t - B_{t-1} \quad (\because CA = \Delta NFA).$$

Suppose the small open economy produces and consumes only one good. There is endowment of output, Y_1 and Y_2, for each period. The endowment goods are perishable. In this economy, the representative household has perfect foresight, and there is neither government nor investment. The discount rate $\beta = 1/(1+\delta)$. The household's problem is to solve

$$\max_{C_1, C_2} u(C_1) + \beta u(C_2) \;\; s.t.\; C_1 + C_2/(1+r) = Y_1 + Y_2/(1+r).$$

The first-order condition to this problem is

$$\frac{\beta u'(C_2)}{u'(C_1)} = \frac{1}{1+r}.$$

If $\delta = r$, the household is going to have a flat consumption profile at $[\overline{C} = (1+r)Y_2 + Y_1]/(2+r)$; if $\delta < r$, then the household is patient and he will have an increasing consumption profile; if $\delta > r$, then the household is impatient and he will have a decreasing consumption profile. In the case that $\delta = r$, we know $CA_1 = 0$ only if $Y_1 = Y_2$; if $\delta = r$ and $Y_1 < Y_2$, then it is optimal for the country to choose a $CA_1 < 0$ path.

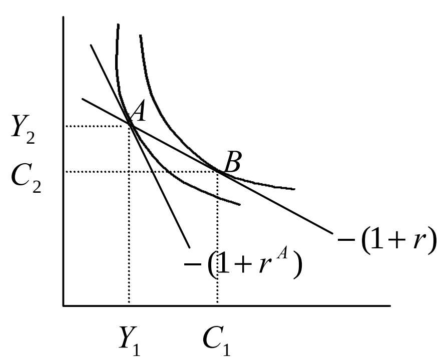

Figure 18

Let the endowment combination to this economy be $\{Y_1, Y_2\}$ at the point A in Figure 18. The Autarchy interest rate r^A could be found by setting $C_1 = Y_1, C_2 = Y_2$ in the first-order condition. We know that r^A should be related to the relative endowment Y_1 / Y_2. Given Y_2, a higher endowment Y_1 implies that future goods becomes relatively expensive so that $P_2 / P_1 \uparrow$ or $r^A \downarrow$. For a small open economy, it is very often the case that $r^A > r$ precisely because the output ratio Y_1 / Y_2 is small relative to the world level. In the meanwhile, we also find a lower r^A associated with a higher β. In a word, we have $r^A = r^A(Y_1 / Y_2, \beta) = r^A(-,-)$.

It is also necessary to distinguish the temporary shocks with permanent shocks. Suppose there are some temporary shocks so that Y_1 rises, given Y_2, then r^A will fall. However, suppose there are some permanent shocks so that Y_1 rises proportional to Y_2, then r^A remains the same.

2. Simple Intertemporal Model with Investment

The first extension to our two-period intertemporal model is to add investment into the model in order to make the model more realistic. This is because investment usually is very volatile and is also a key component in current account.

The present value budget constraint will be

$$Y_1 + Y_2 /(1+r) = (C_1 + I_1 + G_1) + (C_2 + I_2 + G_2)/(1+r).$$

For the moment, let's assume $G_1 = G_2 = 0$, and thus we have

$$C_2 = (Y_2 - I_2) + (1+r)(Y_1 - C_1 - I_1).$$

Moreover, we assume that this small economy has inherited an endowment of capital stock K_1, and the law of capital accumulation is $K_{t+1} = K_t + I_t$ for each period, assuming the depreciation rate is zero. It is common sense that for the two-period model, we have $K_3 = 0$ so that $I_2 = -K_2$ and $K_2 = K_1 + I_1$. We assume the production function for each period follows $Y_t = F(K_t)$ and $F'(K_t) > 0, F''(K_t) < 0$. Plugging these into the budget constraint, we end up with

$$C_2 = F(K_1 + I_1) + (K_1 + I_1) + (1+r)[F(K_1) - C_1 - I_1].$$

If we plug this constraint into the objective function, we have

$$\max_{C_1, I_1}\{u(C_1) + \beta u\{F(K_1 + I_1) + (K_1 + I_1) + (1+r)[F(K_1) - C_1 - I_1]\}\}.$$

The first-order-conditions are of very familiar form as follows

$$u'(C_1) = \beta(1+r)u'(C_2)$$

and

$$F'(K_2) = r.$$

Note that the investment decision is given by the second FOC regardless of the consumption decision. We will soon find that in a large country model where the country's consumption decision has certain impact on the interest rate, the investment decision is closely related with the consumption decision.

Next, let's graphically represent the economy in Figure 19. The horizontal ending point $C_1^{\max}$ of the PPF curve on the C_1 axis satisfies $C_1^{\max} = F(K_1) + K_1$, and the vertical ending point $C_2^{\max}$ of the PPF curve on the C_2 axis satisfies $C_2^{\max} = F[F(K_1) + K_1] + [F(K_1) + K_1]$. The reasoning behind the point $C_1^{\max}$ is the following:

$$C_2 = 0 \ plus \ C_2 = F(K_2) + K_2 \Rightarrow K_2 = 0 \ plus \ K_2 = K_1 + I_1 \Rightarrow I_1 = -K_1 \Rightarrow C_1^{\max} = F(K_1) + K_1.$$

The reasoning behind the point $C_2^{\max}$ is the following:

$$\begin{aligned} C_1 = 0 \ plus \ C_1 = F(K_1) - I_1 \Rightarrow I_1 = F(K_1) \ plus \ K_2 = K_1 + I_1 \Rightarrow K_2 = F(K_1) + K_1 \\ \Rightarrow C_2^{\max} = F[F(K_1) + K_1] + [F(K_1) + K_1] \end{aligned}.$$

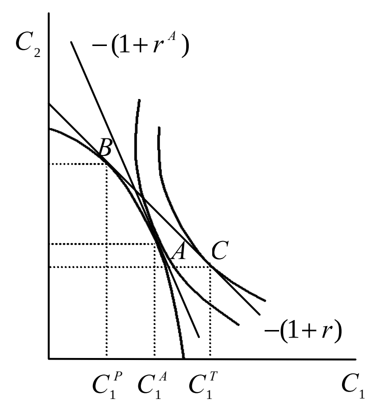

Figure 19

At the autarchy point A, we know the current consumption would be C_1^A, and the current investment would be the difference between the PPF curve horizontal ending point consumption level and the current consumption level, i.e., $I_1 = C_1^{\max} - C_1^A$.

Suppose now we can borrow and lend freely at the world interest rate level $r < r^A$. Some empirical work shows that the consumption point will shift to point C, (This is similar to the results from assuming that the substitution effect dominate the income effect.) whereas the production point would shift to point B. It is apparent that under intertemporal trade we are going to consume more at the current period, $C_1^T - C_1^A$, than the Autarchy point A level. Note further that at production point B, the current investment level is $C_1^{\max} - C_1^P$, i.e., $C_1^A - C_1^P$ units more than the Autarchy point level. Obviously the current account CA is in balance at the Autarchy point A. What about the trade point B then? Clearly the economy is experiencing a trade deficit at the current period so that $CA < 0$. What is the magnitude of the deficit? Recall that one way of interpreting current account is $CA = Y - (C + I + G)$. Remind ourselves that the total income available is not changed due to the exogenous initial capital stock, then the current account deficit must totally attributed to the increase of consumption and investment, given zero government spending. That is, $CA = C_1^T - C_1^P$.

3. Simple Intertemporal Model with Infinite Horizon

The next extension of the intertemporal model is to increase the horizon of the agents. Before we operate the model under the new situation, it is necessary to remind ourselves the way of interpreting current account. We know that for each period s, we have $CA_s = B_{s+1} - B_s = Y_s + rB_s - C_s - I_s - G_s$ and $I_s = K_{s+1} - K_s$. Thus we could interpret the current account as follows $CA_s = \Delta B_s = S_s - I_s = S_s - \Delta K_s \Rightarrow S_s = \Delta B_s + \Delta K_s$. That is to say, the saving at each period is the sum of net foreign assets holding and net capital change.

If we rewrite the expression for CA as

$$B_s = \frac{C_s + I_s + G_s - Y_s}{1+r} + \frac{B_{s+1}}{1+r},$$

we get the intertemporal budget constraint, or the flow budget constraint. If we iterate this forward T periods, we end up with the present value budget constraint

$$(1+r)B_t + \sum_{s=t}^{t+T} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s) = \sum_{s=t}^{t+T} \left(\frac{1}{1+r}\right)^{s-t} (C_s + I_s) + \left(\frac{1}{1+r}\right)^{T} B_{t+T+1}.$$

Without surprising, if we imposing the no-Ponzi-game condition, or the transversality condition $\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^{T} B_{t+T+1} = 0$, we have the following present value budget constraint

$$(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - C_s - I_s - G_s) = 0.$$

This key equation is very often referred to discuss the issue of "solvency" in the following sense. The inherited obligation of the economy is $(1+r)B_t$, and the term $Y_s - C_s - I_s - G_s$ is essentially the trade balance. Therefore, the key equation is saying that the economy should use trade surpluses to pay off the initial debt eventually but this puts nothing restrictive to the current account. The economy may run current account deficit and use the trade surplus to pay off the debt. Empirically people often use this relationship to ask whether a country experiencing a large current account deficit is solvent or not, by calibrating the economy in such an equation.

Next, we are going to discuss a fundamental current account question. First of all, let's denote the permanent level of a variable X as $\tilde{X}$, and the permanent level is in the sense that

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \tilde{X}_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} X_s$$

holds.

Let's further assume $\beta = 1/(1+r)$ so that the optimal consumption profile $\overline{C}_s$ is flat. We know the permanent levels of each variable should satisfy the present value budget constraint as

$$(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (\tilde{Y}_t - \tilde{C}_t - \tilde{I}_t - \tilde{G}_t) = 0.$$

This implies that

$$\overline{C}_t = \tilde{C}_t = rB_t + (\tilde{Y}_t - \tilde{I}_t - \tilde{G}_t).$$

If we combine this relationship with the current account

$$CA_s = Y_s + rB_s - C_s - I_s - G_s,$$

we have the so-called "fundamental current account equation" as

$$CA_t = (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t).$$

This is, the deviation of current income from the permanent income level has a positive relationship with the current account balance. Suppose further there occur a persistent shock to the total factor productivity so that it has different impact on the current income level and the permanent income level, then the change of current account may not be easily determined.

02/23/1999

Subject: Dornbusch Model (I)
Reference: Dornbusch, R. (1983) "Real Interest Rates, Home Goods and Optimal External Borrowing," *Journal of Political Economy*, February

In Dornbusch's paper, the home goods actually stand for nontradable goods. Suppose a small open economy produces tradable and nontradable goods, using fixed capital and internally mobile labor with upper bound of $\bar{L}$. The *numeraire* good in this model is tradable, and the relative prices at period t are defined as $\rho_t \equiv P_{N,t}/P_{T,t}$. The prices of both tradable goods and nontradable goods could change over time, although the tradable prices are determined outside this economy. Suppose the world interest rate is fixed at r^*. We could think of the relationship between the relative prices and the domestic real interest rate r: suppose $\rho_{t+1} > \rho_t$, then given P_T, it says that $P_{N,t+1} > P_{N,t}$. Since future goods are relatively expensive, that means the domestic real interest rate must be lower than the world level, i.e., $r < r^*$. Of course, we are going to verify this relationship shortly.

Denote as y_t the endowed output in terms of *numeraire* at period t, and let b_0 be the initial debt stock. Further define the discount factor as $D \equiv 1/(1+\delta)$, and the gross world interest rate as $R \equiv 1 + r^*$. Let's assume the infinitely living representative household has the following objective:

$$\max_{C_T, C_N} \sum_{t=0}^{\infty} D^t U_t(C_{T,t}, C_{N,t}) \text{ s.t.} \sum_{t=0}^{\infty} (C_{N,t}\rho_t + C_{T,t} - y_t) R^{-t} + b_0 = 0 \text{ and } \lim_{t\to\infty} b_t R^{-t} = 0.$$

We further assume that the utility function takes the form of CRRA as

$$U_t = \frac{C_t^{1-\theta}}{1-\theta}, \text{ where } C_t \equiv C_{T,t}^{a} C_{N,t}^{1-a}.$$

Here C_t is the index of total consumption, θ is the coefficient of relative risk aversion and also the inverse of intertemporal elasticity of consumption, and a is the consumption share of the tradable goods. If $a = 1$, then the nontradable goods won't matter at all, then we should be able to get the result of $r = r^*$. We also know that as $\theta \to 0$, the utility will become straight; as $\theta \to 1$, the utility will become log utility; and as $\theta \to \infty$, the utility will take the form of Leontief.

First-order conditions to the optimization problem are

$$U_t^N / U_t^T = \rho_t \quad \text{and} \quad U_t^N / U_{t+1}^N = DR(\rho_t / \rho_{t+1}).$$

If we substitute the marginal utilities of the CRRA function into the second condition, we end up with the CC schedule

$$\frac{C_t}{C_{t+1}} = \left(\frac{1+r^*}{1+\delta}\right)^{-1/\theta} \left(\frac{\rho_{t+1}}{\rho_t}\right)^{(1-a)/\theta}.$$

Suppose $\rho_t = \rho_{t+1}$, then $r^* > \delta$ would imply an increasing consumption profile and $r^* = \delta$ would imply a flat consumption profile.

We could rewrite the CC schedule equation as follows

$$\frac{C_t}{C_{t+1}} = \left[\left(\frac{1}{1+\delta}\right)(1+r^*)\left(\frac{\rho_t}{\rho_{t+1}}\right)^{1-a}\right]^{-1/\theta} = \left[\frac{1+r}{1+\delta}\right]^{-1/\theta}, \text{ where } r \equiv (1+r^*)\left(\frac{\rho_t}{\rho_{t+1}}\right)^{1-a} - 1.$$

Here the real interest rate r is called the home goods real interest rate or home consumption real interest rate. If $a = 1$, then we have $r = r^*$; if $\rho_t = \rho_{t+1}$, then we also have $r = r^*$. We know that θ would determine the intertemporal smoothness of consumption; a higher θ implies higher risk aversion and thus smoother consumption. Suppose for the moment that we have $\delta = r^*$ and $0 < a < 1$. Let's graphically present the CC schedule in Figure 20.

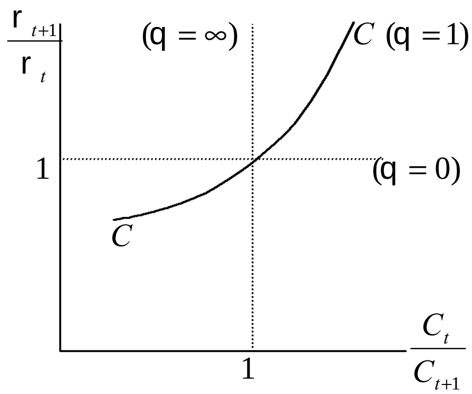

Figure 20

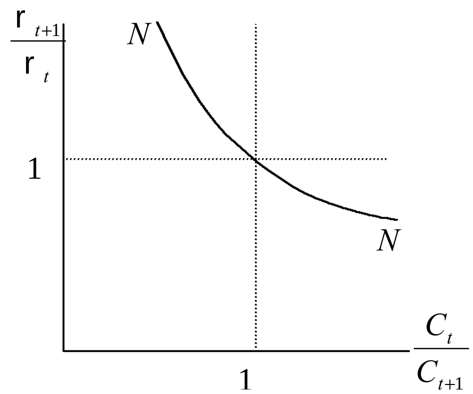

Figure 21

First of all, we know that when $\rho_{t+1} = \rho_t$ we have $C_t = C_{t+1}$ so that the CC schedule would go through the point $(1,1)$. Let's consider the case where $\theta = 1$. Suppose there is an increase in future relative prices, we know that the future nontradable goods would be more expensive given the tradable goods prices so that the home goods real interest rate must be lower than the world level, i.e., $r < r^* = \delta$. This would imply a declining consumption profile so that we have upward sloping CC schedule under the case of $\theta = 1$. As θ becomes smaller, the representative becomes less risk averse so that he would not save enough today and have a declining consumption profile. That is, the CC schedule would become flatter. At the extreme case where $\theta = 0$, we know the CC schedule would become flat corresponding to perfect intertemporal substitution of consumption; at the other extreme case where $\theta = \infty$, we know that CC schedule would become vertical corresponding to the perfect smoothing of consumption.

To determine the equilibrium point in this economy, we have to utilize the market clearing condition in the nontradable goods market. Suppose the demand for and supply of nontradables are given by $C_{N,t} = \phi C_t \rho_t^{-a}$ and $q_{N,t} = \bar{q}_t \rho_t^{\varepsilon}$, respectively. $\bar{q}_t$ is the productivity shocks to the nontradable goods production and ε is the supply elasticity in response to the changes in relative prices. The market clearing condition in the nontradable goods sector implies that

$$\phi C_t \rho_t^{-a} = \bar{q}_t \rho_t^{\varepsilon} \Rightarrow \rho_t = \left(\frac{\bar{q}_t}{\phi C_t}\right)^{-1/(a+\varepsilon)}.$$

This is saying that a positive shock to the production of nontradable goods would make the relative prices decline, which makes sense. If we compare the relative prices in two periods, we could have the NN schedule as follows,

$$\frac{\rho_{t+1}}{\rho_t} = \left(\frac{C_t}{C_{t+1}} \frac{\bar{q}_{t+1}}{\bar{q}_t}\right)^{-1/(a+\varepsilon)}.$$

Suppose the productivity remains the same across periods, then we find that the NN schedule also go through the point $(1, 1)$. Suppose the representative household is going to consume more current goods, then the current nontradable goods becomes more expensive relative to future ones so that the NN schedule is downward sloping as in Figure 21.

If we want to determine how much to consume currently, then we have to go back to the budget constraint and find out the output and initial debt condition etc. This analysis will be ignored here. Suppose there is an initial debt, it is obviously that we should have a CA surplus to pay off the debt. A few experiments will be considered in the next lecture to assess the responses of CA to the productivity shocks.

02/25/1999

Subject: Dornbusch Model (II)
Reference: Dornbusch, R. (1983) "Real Interest Rates, Home Goods and Optimal External Borrowing," *Journal of Political Economy*, February

<u>Experiment #1</u>

Suppose there is an expected permanent productivity shocks in the nontradable sector, i.e., $\bar{q}_{t+i} > \bar{q}_t, \forall i > 0$. What will happen to the current account at present? One might think that since the productivity is going to increase permanently, the increased permanent income must imply that we have to consume more now while the current income hasn't changed yet so that $CA_t < 0$. However, there is also another effect intuitively: since the productivity is going to increase, the future nontradable goods become cheaper relative to today's nontradable goods so that the representative household has to consume less today, corresponding to the higher effective interest rate. This would imply that $CA_t > 0$. Just like the income effect and substitution effect corresponding to the price change, we have dual effects here also, and it is really ambiguous whether the economy would experience a current account deficit or surplus.

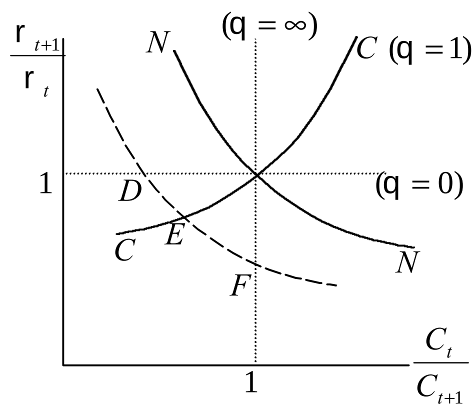

Figure 22

Graphically, suppose the economy is at the equilibrium point $(1,1)$ initially and there is no initial debt and the current account is in balance. The expected permanent productivity shock implies that future goods will be cheaper relative to current goods so that the NN schedule would shift downward. Depending on specific CC schedule, the economy could end up with one of D, E, F points corresponding to $\theta = 0, \theta = 1, \theta = \infty$ cases, respectively.

Suppose that the representative household is a perfect smoother $(\theta = \infty)$, i.e., s/he is highly sensitive to the permanent income. The higher permanent income associated with the permanent shock would imply that consumption at every period, including period t, would increase accordingly. Since there is no change to the production in the current period t, a higher current consumption level implies the current account deficit, i.e., $CA_t < 0$. (Or in details, the higher current consumption implies a higher demand for nontradable goods and higher current price of nontradable relative to tradable. Therefore, the consumption of tradable would be higher in response to both a higher current consumption level and higher relative price of nontradable vs. tradable.)

Suppose that the representative household is a perfect substituter $(q = 0)$, then s/he would be highly sensitive to the effective real interest rate. The expected permanent shock implies that future goods will be cheaper relative to current goods so that r_{t+1} would fall, i.e., a higher effective interest rate. Then the response of a lower current consumption level would imply a lower demand for nontradable goods and thus lower price of nontradable relative to tradable. Therefore, the consumption of tradable would decline in response to both the lower current consumption level and the lower relative price of nontradable vs. tradable. That is to say, the economy is experiencing a current account surplus at the current period. Moreover, we notice that the fall of price of nontradable vs. tradable implies a higher future goods price relative to current. This process would continue until in the equilibrium it returns to the original level. We can show by mathematical reasoning that the case of $q = 1$ corresponds to the current account in balance because the consumption smoothing impact and substitution impact cancel out with each other.

We could also formally prove the arguments above in the following way. If we are to draw the similar diagram for the economy in period $t+1, t+2, \ldots$, it will not be surprising to find that the economy would go back to the equilibrium point $(1, 1)$. The market clearing condition on the nontradable goods sector implies that $Y_N = C_N$ holds at period t, and $Y'_N = C'_N$ holds at period $t+i$ $(i > 0)$. Note that we are going to use a prime to denote the future variable at the equilibrium level from now on. It is obvious that we need to find out the current equilibrium level of tradable goods consumption in order to determine the current account. In the steady states, if we apply the equilibrium conditions for nontradable goods, the present value budget constraint would reduce to

$$(Y_T - C_T) + \sum_{i=1}^{\infty} \left[\frac{Y'_T - C'_T}{(1+r^*)^i} \right] = 0 \Rightarrow (Y_T - C_T) + (Y'_T - C'_T)/r^* = 0 .$$

The optimization problem of the representative household is then

$$\max_{C_T} \frac{\left[C'^{a}_T \, Y'^{(1-a)}_N \right]^{1-q} /(1+d) + \left[C^{a}_T Y^{(1-a)}_N \right]^{1-q}}{1-q} \text{ s.t. present value budget constraint} .$$

The solution to this problem is

$$C_T = Y_T \left[\frac{k + r^* k}{1 + r^* k} \right], \text{ where } k \equiv \left(\frac{Y_N}{Y'_N} \right)^{a}, \text{ and } a = \frac{(1-q)(1-a)}{1-a(1-q)} .$$

Now we have the situation regarding current account as follows.

(1) $q = 1 \Rightarrow C_T = Y_T \Rightarrow CA_t = 0$

(2) $q < 1 \Rightarrow C_T < Y_T \Rightarrow CA_t > 0$ (substituter)

(3) $q > 1 \Rightarrow C_T > Y_T \Rightarrow CA_t < 0$ (smoother)

We could also do some experiment like the effects of a permanent shock to the tradable productivity, or the effects of changes in demand, etc. We are going to consider another interesting experiment regarding effects of changes in the world interest rate.

<u>Experiment #2</u>

Suppose there is no initial debt in the economy, and there occurs a temporary increase of world interest rate at period t so that $r_t^* > r^* = r_{t+i}^*$, $(i = 1, 2, \ldots)$.

Apparently, there is no impact on the NN schedule and the CC schedule would shift toward the left in that a higher interest rate would encourage people to save more today. Alternatively speaking, a higher interest rate would imply the current nontradable goods relatively expensive. Graphically in Figure 23, if the CC schedule is under the case of $q = 1$, then the economy would move from the initial equilibrium point A to point B. Under the case of $q = \infty$, however, the CC schedule would not shift at all in that the change of interest rate is only temporary, and there is nothing changing in the permanent income so that the perfect smoother wouldn't change it's current and future consumption. Under the case of $q = 0$, the CC schedule won't shift either in that the relative prices remain the original level. Therefore, the economy would stay at the same equilibrium point A corresponds to the latter cases.

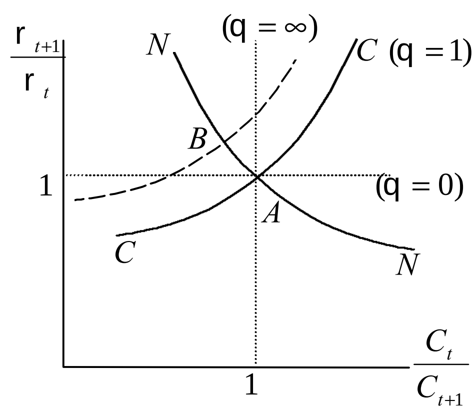

Figure 23

Of course we could formally prove the reasoning above as well. Under the case of certain substitution, the situation of current account turns to depend upon the initial debt/assets condition. Suppose there exists some initial debt in the economy. The higher world interest rate would imply a higher interest payment and thus lower permanent income. Therefore there must be a current account surplus at the current period to finance the future current account deficit. Overall, the presence of a nontradable sector tends to dampen the effect of international interest rate changes on the consumption profile and the trade balance.

03/02/1999

Subject: Frankel and Razin Fiscal Policy Model (I)
Reference: Frankel, J. and A. Razin (1996) *Fiscal Policies and Growth in the World Economy*, 3rd ed. Cambridge, MIT press. Chapter 9-10, pp. 311-75

It seems useful to discuss the Metzler diagram before we discuss the fiscal policy model because this diagram gives us a very good intuition for two-country model. In a typical two-country model, we assume that the capital market is totally open to both countries so that each of them could borrow freely at the same world interest rate r. Then the current account in the home country could be represented by $CA = S(r,\mathrm{x}) - I(r,\mathrm{g}) = S(+,?) - I(-,?)$, where x is country specific factors other than interest rate that determines the domestic saving, and g stands for country specific factors determines the domestic investment. If we put a star on every variable for the rest of the world (presumably another aggregate country), then we have

$$CA^* = S^*(r,\mathrm{x}^*) - I^*(r,\mathrm{g}^*) = S^*(+,?) - I(-,?) \text{ and } CA + CA^* = 0 .$$

Suppose the home country is a developed one and it has a higher autarchy interest rate than the rest of the world in that the developing countries tend to have more future goods relative to today so that they have a higher autarchy interest rate. In the following figure, it is clear that the endogenously determined world interest rate lies between these two autarchy interest rates.

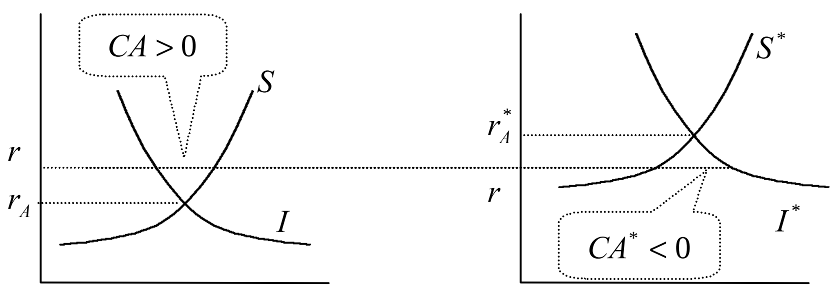

Figure 24

Now let's talk about the Frankel and Razin model in the following setup. There are two large countries that produce only traded goods. The capital market is open so that the world interest rate r_t would be endogenously determined. (It would turn out that the world interest rate acts as the transmission mechanism for fiscal policy across two countries.) For simplicity, we assume that each country would be endowed with some output in the beginning of each period.

Let's consider the situation in the home country at first and then put stars on each variable to get corresponding one for the aggregate economy standing for the rest of the world. There is no investment in this model so that we could interpret current account as the sum of private saving and government saving, i.e., $CA = S_p + S_g$. The total amount of debt could also be decomposed into private part and government part, i.e., $B = B_p + B_g$. We assume that the government would has infinite horizon so that we have the following present value budget constraint,

$$\sum_{s=0}^{\infty}\left[\prod_{v=0}^{s-1}\left(\frac{1}{1+r_v}\right)(T_s - G_s)\right] = B_{g0} .$$

If we define $\alpha_s = \prod_{v=0}^{s-1}\left(\frac{1}{1+r_v}\right)$, then we have $\sum_{s=0}^{\infty}\alpha_s(T_s - G_s) = B_{g0}$.

On the consumer side, we assume that each agent would have a probability of γ to survive to the next period. By definition, we have $0 \le \gamma \le 1$, and the case of $\gamma = 1$ corresponds to infinite horizon for the consumer, and the case of $\gamma = 0$ corresponds to one horizon for the consumer.

Since the consumers are subject to death at certain probability, it is not possible for the consumers to act as lender. In order to find out the real interest rate in this world, we need to introduce some outside life insurance company. This company would charge each unit of good that consumer borrows a premium p so that in case that the consumer would die next period, the insurance company would pay off the debt the consumer owns. A fair price of the premium would make the expected profit of the insurance company be zero. That is, for the case of survival, one unit of loan would make the company $(1+r)p\gamma$ units of profit, for the case of death, one unit of loan would make the company $(1+r)(1-\gamma)$ units of debt, and thus we have the fair price as $(1-\gamma)/\gamma$.

To get the effective interest rate, suppose the insurance company loaned the household one unit today. How much should it ask back in the next period? It turns out to be $(1+r)/\gamma$ so that its sure income in the next period, $[(1+r)/\gamma]\gamma = (1+r)$, will cancel out with its sure debt in the next period, $(1+r)$. Therefore, the effective gross interest rate in this economy would be $(1+r)/\gamma$. We know that the interest discount factor is just the inverse of the effective gross interest rate; therefore, when $\gamma \in (0,1)$, the effective discount factor is $1/[(1+r)/\gamma] = \gamma/(1+r)$. Define $R \equiv 1/(1+r)$, then the effective discount factor is $R\gamma$.

For simplicity, let's normalize the cohort size to 1 so that the total size of cohort at age of s would be γ^s. For each period t, the population size is then given by

$$\sum_{s=-\infty}^{t}\gamma^{t-s} = 1/(1-\gamma).$$

This is to say, we have a constant population each period in this economy. Suppose further that the consumer's utility function is of logarithmic each period, and the discount factor is δ. If we work out the individuals' optimization problem, we could get the following form of aggregate consumption for each period t,

[1] $C_t = (1-\gamma\delta)W_t$,

where W_t is the aggregate wealth level in the home country for period t. This makes sense in that a higher discount rate would imply a lower discount factor for an impatient agent and thus higher current consumption, and that a lower survival rate would imply more current consumption. Similarly, we have the aggregate consumption for the rest of world as follows,

[2] $C_t^* = (1-\gamma\delta)W_t^*$,

where W_t^* is the aggregate wealth level for the rest of the world.

Our next topic is about the wealth level. Let W_0 be the discounted value of the household wealth at period 0. Then we have

$$W_0 = \sum_{s=0}^{\infty} \gamma^s \alpha_s (Y_s - T_s) - B_{p0}.$$

If we use the relations $B_0 = B_{p0} + B_{g0}$ and the government present value budget constraint, we can further express the wealth level above as $W_0 = \sum_{s=0}^{\infty} \gamma^s \alpha_s Y_s - B_0 + \left[B_{g0} - \sum_{s=0}^{\infty} \gamma^s \alpha_s T_s \right]$. We could have some interesting interpretation regarding two extreme cases for the survival rate. We have the similar relation for the rest of world as follows,

$$W_0^* = \sum_{s=0}^{\infty} \gamma^s \alpha_s (Y_s^* - T_s^*) - B_{p0}^*.$$

Next we try to work out all the relationships in the two economies in terms of two periods. For the home country, we have the following

$$\sum_{s=0}^{\infty} \gamma^s \alpha_s (Y_s - T_s) = (Y_0 - T_0) + \frac{\gamma R}{1 - \gamma R}(Y - T),$$

where R is the certainty equivalent interest factor, i.e., the inverse of certainty equivalent gross interest rate, and $Y_1 = Y_2 = \cdots \equiv Y, T_1 = T_2 = \cdots = T$. Using the relation above, we have the domestic wealth level as

$$W_0 = (Y_0 - T_0) + \frac{\gamma R}{1 - \gamma R}(Y - T) - B_0 + B_{g0}.$$

It is important to note that the interest factor is endogenously determined here, and there is a positive relationship between W_0 and R if $Y > T$. Using the relation $B_0 = -B_0^*$, we have the parallel wealth level for the rest of the world as

$$W_0^* = (Y_0^* - T_0^*) + \frac{\gamma R}{1 - \gamma R}(Y^* - T^*) + B_0 + B_{g0}^*.$$

03/04/1999

Subject: Frankel and Razin Fiscal Policy Model (II)
Reference: Frankel, J. and A. Razin (1996) *Fiscal Policies and Growth in the World Economy*, 3rd ed. Cambridge, MIT press. Chapter 9-10, pp. 311-75

Let's briefly review what we have discussed in last lecture. We use a subscript $_0$ to denote variables at the current period, and no subscript to denote variables in the future with certainty equivalence. In the government side, we have the following budget constraints for the home country and the rest of world,

$$[1]\,(T_0 - G_0) + \frac{R}{1-R}(T - G) = B_{g0}$$

and

$$[2]\,(T_0^* - G_0^*) + \frac{R}{1-R}(T^* - G^*) = B_{g0}^* .$$

Let's assume at the moment $T_0^* = G_0^*, T^* = G^*$ and $B_{g0}^* = 0$ for simplicity. On the households' side, we have the following relationships:

$$[3]\; C_t = (1-\gamma\delta)W_t$$

$$[4]\; C_t^* = (1-\gamma\delta^*)W_t^*$$

$$[5]\; W_0 = (Y_0 - T_0) + \frac{\gamma R}{1-\gamma R}(Y - T) - B_0 + B_{g0}$$

$$[6]\; W_0^* = (Y_0^* - T_0^*) + \frac{\gamma R}{1-\gamma R}(Y^* - T^*) + B_0 + B_{g0}^* .$$

Note that W_0 and W_0^* here are the present values of wealth level for those currently alive.

To close the model, we need to develop goods market equilibrium condition. Here basically we have two goods market, one for current goods and the other for future goods. The demand for current goods in the private sector consists of two parts: one comes from the current alive at the home country, and the other comes from the current alive at the rest of the world. Therefore, the demand for current goods could be written as $(1-\gamma\delta)W_0 + (1-\gamma\delta^*)W_0^*$. The supply of current goods in the private sector is just the sum of goods available for the private sector at home and in the rest of the world, i.e., $(Y_0 - G_0) + (Y_0^* - G_0^*)$. Therefore, we have the following present goods market equilibrium condition, called PP schedule,

$$[7]\; (1-\gamma\delta)W_0 + (1-\gamma\delta^*)W_0^* = (Y_0 - G_0) + (Y_0^* - G_0^*) .$$

In terms of the equilibrium condition for the future goods, it is a little bit complicated in that we should consider the agents that are not born yet. For those people currently alive, their total wealth levels are W_0 and W_0^*, and we know they only consume $(1-\gamma\delta)W_0$ and $(1-\gamma\delta^*)W_0^*$ at the present. Hence the demand for future goods from the people who are currently alive and will be alive in the future is simply $\gamma\delta W_0 + \gamma\delta^* W_0^*$. Next we are going to determine how much the people unborn yet would demand for the future goods. First of all, these people will use all their disposable income to consume future goods since we consider only two periods here. We know that the population size is $1/(1-\gamma)$ in each future period, and the disposable income for each cohort is $Y - T$ at the value of each period. The *per capita* disposable income is then $(1-\gamma)(Y-T)$ in each future period. The period 0 value of the *per capita* disposable income at each period will be

$$\frac{1}{1-\gamma R}(1-\gamma)(Y-T),$$

considering the survival rate of γ. Note that we don't use $\gamma R/(1-\gamma R)$ because people won't be born until the very period next to the current one. Remember that in this calculation we use $Y-T$ at the value of each future period, and we also need to discount this back to the period 0 value. Therefore, the period 0 value of the *per capita* disposable income at all future periods will be

$$\frac{R}{1-R}\frac{(1-\gamma)(Y-T)}{1-\gamma R}.$$

This is also the demand for future goods from all the people unborn yet at the home country. The total demand for future goods from those yet unborn in the private sector would be

$$\frac{R}{1-R}\frac{(1-\gamma)(Y-T)}{1-\gamma R}+\frac{R}{1-R}\frac{(1-\gamma)(Y^*-T^*)}{1-\gamma R}.$$

The supply of future goods in the private sector would be apparently

$$\frac{R}{1-R}\left[(Y-G)+(Y^*-G^*)\right].$$

Therefore, the future goods clearance condition is the following, called FF schedule,

$$[8]\ \gamma\delta W_0+\gamma\delta^* W_0^*+\frac{R}{1-R}\frac{(1-\gamma)(Y-T)}{1-\gamma R}+\frac{R}{1-R}\frac{(1-\gamma)(Y^*-T^*)}{1-\gamma R}=\frac{R}{1-R}\left[(Y-G)+(Y^*-G^*)\right]$$

Essentially we have the system of four equations, namely [5], [6], [7], [8], to solve three unknowns, $W_0, W_0^*,$ and R. According to Walras law, one of the equations is redundant and here we chose to use only [6], [7] and [8]. The exogenous variables in this system could be classified as two groups: one is policy variables, such as $G_0, T_0, G, T, G_0^*, T_0^*, G^*, T^*$, subject to the government budget constraints, and the other is endowment variables, such as $Y_0, Y, Y_0^*, Y^*, B_0, B_{g0}, B_{g0}^*$.

Ideally we want to graphically depict the relationship between these three unknowns in Figure 25.

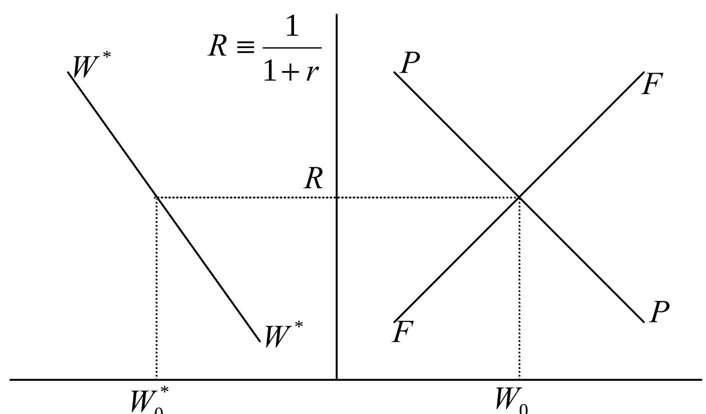

Figure 25

Throughout our discussion, we assume that $Y > T$ and $Y^* > T^*$. Equation [6] implies that $W_0^* = W_0^*(R) = W_0^*(+)$, and thus the wealth schedule W^*W^* is upward sloping. Equation [7] implies that $PD(W_0, R) = PS$, where PD stands for the demand for current goods and PS stands for the supply of current goods. Furthermore, R enters PD because of $W_0^*(R)$, and $PD(W_0, R) = PD(+,+)$. Suppose there is a point satisfying the current goods clearance condition, and we increase W_0 a little bit. The excess demand for current goods would call on the decline of interest factor R to offset the gap between demand and supply through the decline of W_0^* and restore the market clearance. That is, the PP schedule is downward sloping. Similarly equation [8] implies that $FD(W_0, R) = FS(R)$, and $FD(+,+) = FS(+)$. Suppose there is a point satisfying the future goods clearance condition, and we increase R a little bit, then what we should do to restore the market clearance again? Intuitively, wealth level in each period would go up corresponding to a higher R, but the increase of wealth level has very different impact on the supply and demand side. The supply of current goods won't change and all the impact of a higher wealth level on supply is to have a higher supply of future goods. Since demand for both current goods and future goods would go up, the wealth effect on the demand side would be split between current goods and future goods. Therefore, we know the impact of a higher R would be an excess supply of future goods, and thus we have to increase the W_0 level to offset the excess supply to restore the future goods clearance.

Experiment #1: Current Taxes Cut

Now let's do an experiment of fiscal policy. Suppose the home country cuts taxes today without changing current and future government spending. The government budget constraint [1] implies that

$$dT_0 = -\frac{R}{1-R} dT,$$

or a taxes cut today $(dT_0 < 0)$ will be financed by more future taxes $(dT > 0)$. From equation [6], we know there is nothing happen to the W^*W^* schedule since what's happening is within the home country. From equation [7], we know the PP schedule will not shift. From equation [8], we know that higher future taxes imply lower future disposable income with certainty equivalence, or lower demand for future goods. In order to rebuild goods clearance, we need to subsidize more wealth W_0 at each of the current R; that is, the FF schedule should shift towards the right as in Figure 26.

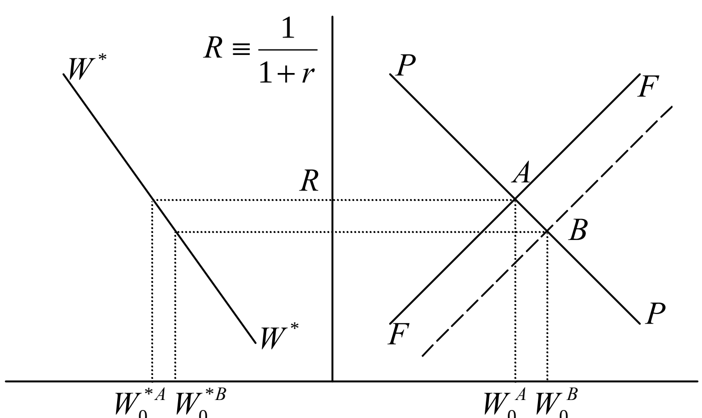

Figure 26

Graphically from Figure 26, we know the impact of the taxes cut in the home country is a decline in R, a higher domestic wealth level W_0 and a lower wealth level W_0^* for the rest of the world. For situation like this, we normally say there is a negative transmission to the rest of the world associated with the higher world interest rate. There are three ways of intuitively interpreting why there is a higher world interest rate associated with the tax cut in the home country. Firstly, because the households realize that they are going to pay higher taxes in the future anyway, we should have a higher world interest rate to allow people to save more at present. However, if $g = 1$ so that both households and government would live infinitely, then the tax cut wouldn't matter in the households' viewpoint and there will be no change in the world interest rate. Secondly, nothing changed in the endowment so that the total disposable income for the households remains the same. Then the taxes cut implies lower future disposable income and thus lower demand for future goods. The shift from demand for future goods to demand for current goods while the system is still in equilibrium implies that the future prices must be lower relative to current ones, which means a higher interest rates. Lastly, we could interpret the increase in world interest rate in terms of "transfer problem". When we are talking about the impact of transferring wealth from one group to another group, the correct way of tackling the problem is to compare the marginal propensity to consume in two groups. In our case, a taxes cut is equivalent to transfer wealth from people in the future to people at present. We know that the marginal propensity to consume current goods for people in the future is zero, and the marginal propensity to consume current goods for people at present is obviously positive. Therefore, we are transferring wealth from a group with lower marginal propensity to consume to a group with higher propensity to consume and thus the price of present goods will rise relative to future goods, which means a higher interest rate.

03/13/1999

Subject: IS-LM-BP Model (I)
Reference: Obstfeld, M. and K. Rogoff (1996) *Foundations of International Macroeconomics*, Cambridge: The MIT Press, Chapter 8, p.p. 511-30

Recall that three basic accounting identities have been discussed in the very first lecture. They are: $Y = C + I + G + (X - M)$, $CA + KA = \Delta R$, and $DC + M = R \Leftrightarrow \Delta DC + \Delta M = \Delta R$. Let's define the nominal exchange rate as $S = US\$ / JPY$. Here the home country is US and the rest of the world is Japan. The real exchange rate is defined as $q = SP^* / P$.

Before introducing money into our discussion, it is useful to review a few key relationships. The first one is the "Law of One Price." This is to say, for the i^{th} individual goods, $P_i = SP_i^*$ holds. The second key relationship is the "Purchasing Power Parity." There are two versions of PPP; the absolute PPP states that $P = SP^*$ for the total basket of goods while the relative PPP states that $\hat{p} = \hat{s} + \hat{p}^*$, or $\hat{s} = \hat{p} - \hat{p}^*$. The hat above variables stands for the percentage changes. Literally the definition of real exchange rate implies $\hat{q} = \hat{s} + \hat{p}^* - \hat{p} \Rightarrow \hat{s} = \hat{q} + \hat{p} - \hat{p}^*$. The relative PPP holds only if $\hat{q} = 0$. However, these relationships are expected to hold at most in the long-run while the short-run deviation from PPP is very obvious.

1. A Simple Monetary Model

Suppose the domestic money market equilibrium condition is $M / P = L^d$, where L^d is the liquidity money demand. Then in steady-state growth rate level, we have $\hat{m} - \hat{p} = \hat{L}^d \Rightarrow \hat{p} = \hat{m} - \hat{L}^d$. Similarly in the foreign country, we have $\hat{p}^* = \hat{m}^* - \hat{L}^{d*}$. Suppose in the exchange rate market, $\hat{q} = f(X)$, where X is the determinant real variables for the steady-state growth rate of real exchange rate. Therefore, the definition of the real exchange rate implies

$$\hat{s} = f(X) + (\hat{m} - \hat{m}^*) - (\hat{L}^d - \hat{L}^{d*}).$$

This is, in the long run, there are primarily three groups of determinant factors for the nominal exchange rate: the real determining variables, the difference in money growth rates and the difference in growth rates of real liquidity demand.

2. IS-LM-BP Model

In a small open economy, suppose the prices and wages are sticky. Our model would depend upon whether this economy adopts fixed or flexible exchange rate regime as well as whether it allows full capital mobility. As one variant of the IS-LM-BP model, the Mundell-Fleming model deals with a small open economy with full capital mobility.

The IS curve is very simple as follows,

$$\begin{aligned} Y &= C(Y; C_0) + I(i; I_0) + G_0 + NX(SP^* / P, Y; X_0, M_0) = (+;...) + (-;...) + (+,-;...) \\ &= Y(i, SP^* / P;...) = (-,+;...). \end{aligned}$$

The variables after the semi-colon stand for exogenous variables. The $IS(S)$ curve is downward sloping in the plane of (i,Y) and will shift to the right corresponding to a higher nominal exchange rate level S.

The LM curve can be characterized as,

$$M/P = (DC+R)/P = L(i,Y;L_0) = L(-,+;...)\,.$$

Let L_0 stand for tastes etc. exogenous variable. Note that the LM curve is not a function of the nominal exchange rate. Even though there is certain impact of S on R, but it is only of second-order impact and we ignore it here. It is apparent that the LM curve should slope up.

The BP curve could be characterize as,

$$NX(SP^*/P,Y;X_0,M_0) + KA(i-i^*;KA_0) = (+,-;...) + (+;...) = \Delta R$$
$$\Rightarrow BP(SP^*/P,Y,i-i^*;...) = (+,-,+;...) = \Delta R = 0.$$

Note that we use the approximation that $CA \approx NX$. For the case of full capital mobility, the existence of slight difference between interest rates across countries would induce massive capital motion. Thus, the condition for balance of payment to be in balance is that the interest rate difference under this case should be zero, i.e., the BP curve is flat. For the case of no capital mobility, assume that the capital account is zero so that we have a vertical BP curve in the (i,Y) plane. For the case with certain mobility of capital we have an upward sloping curve and a higher capital mobility implies a flatter slope of the BP curve. At the third case, any point above the BP curve implies that there is a higher capital account balance than necessary to keep the balance of payment in balance and thus a lower interest rate is called for to bring it back to balance.

These three relations can be graphically presented in the following two figures. Figure 27 stands for the case of perfect capital mobility and Figure 28 stands for the case of low capital mobility.

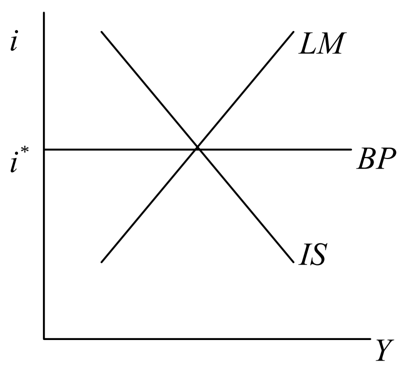

Figure 27

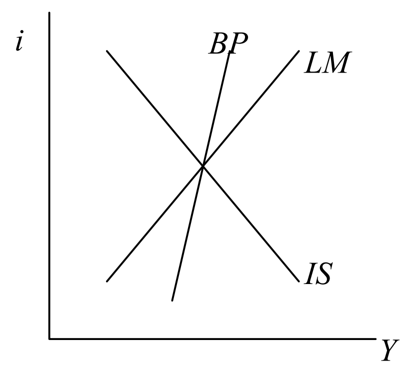

Figure 28

03/25/1999

Subject: IS-LM-BP Model (II)
Reference: Obstfeld, M. and K. Rogoff (1996) *Foundations of International Macroeconomics*, Cambridge: The MIT Press, Chapter 8, p.p. 511-30

We have derived the following three relationships:

[1] $Y = Y(i, SP^* / P) = Y(-,+)$

[2] $\frac{M}{P} = \frac{DC + R}{P} = L^d(i, Y) = L^d(-,+)$

[3] $BP(SP^* / P, Y, i - i^*) = BP(+,-,+) = \Delta R = 0$

Assume that both prices P, P^* and nominal wages are sticky in the short-run. Let's do a few policy experiments as follows.

1. Fixed Exchange Rate Regime

Under fixed exchange rate regime, the reserve has to change to reach the balance of BP account. This doesn't have any shifting impact on the IS or BP curve but LM curve. Also note that when there is perfect capital mobility, people would be extremely sensitive to the change in interest rate; on the other side, when there is zero capital mobility, people would pay attention to only the change of output. The three equations above simultaneously determine the levels of $Y, i,$ and R. Keep in mind that the location of the economy is determined by the IS and LM curves, and that we read the BOP from the BP curve.

Experiment 1.1 Fiscal Expansion (Increase in Government Expanding)

Corresponding to a fiscal expansion, the IS curve would shift to the right. The equilibrium point would move from point A to point B if the economy is a closed one, and there will be a higher output and interest rate. For the case of a small open economy, the higher interest rate would imply an improvement of BP account, but the higher income level would imply a deterioration of BP account. Depending upon the extent of capital mobility, the economy would end up with different equilibrium.

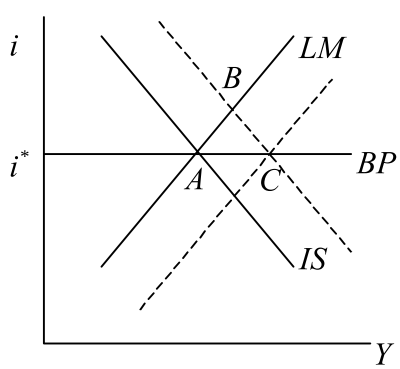

Figure 29

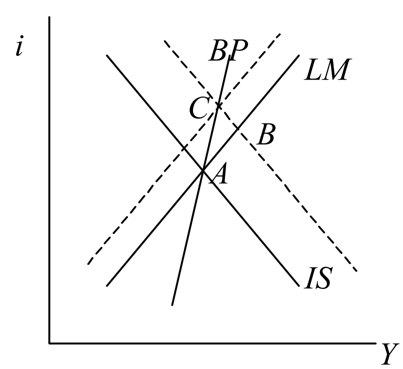

Figure 30

For the case of perfect mobile capital, people are so sensitive to the interest rate that the impact on improvement of BP account dominates and thus the reserve would increase. The improvement of

BP account means that point *B* must locate above the *BP* curve. The higher reserve level means more money supply and thus the *LM* curve shifts to the right until the intersection point *C* between the *BP* curve and *IS* curve. Therefore, a fiscal expansion in a small open economy with perfectly mobile capital under fixed exchange rate regime would increase the output, without changing the interest rate and the *BP* account balance. Its impact on the output is stronger than in the closed economy whereas its impact on the interest rate is weaker. This is precisely because a lower interest rate is necessary to suppress the otherwise *BP* account surplus associated with higher reserve level, i.e., there is an expansionary monetary policy reinforcing the fiscal expansion.

For the case of low capital mobility, people pay more attention to the increase of income rather than the increase of interest rate so that there will be a deterioration of *BP* account, which induces the fall of reserve level. The deterioration of *BP* account means that point *B* must locate below the *BP* curve, i.e., we should have a *BP* curve steeper than the *LM* curve. The lower level of reserve means the shrinking of money supply and thus the *LM* curve shifts to the left until the intersection point *C* between the *BP* curve and *IS* curve. Therefore, a fiscal expansion in a small open economy with low capital mobility under fixed exchange rate regime would increase the output, but to an extent less than in the closed economy, and increase the interest rate, but to an extent more than in the closed economy. Once again, this is because a higher interest rate is necessary to suppress the otherwise *BP* account deficit associated with lower reserve level, i.e., there is a monetary contraction offsetting the fiscal expansion.

<u>Experiment 1.2 Monetary Expansion (Increase in Domestic Credit)</u>

Corresponding to the monetary expansion, the *LM* curve would shift to the right. The equilibrium point would move from point *A* to point *B* if the economy is a closed one, and there will be a higher output and lower interest rate. For the case of a small open economy, the lower interest rate would imply a deterioration of *BP* account, and the higher output level would do the same. That is, regardless of the extent of capital mobility, the lower reserve level would force the *LM* curve shift back to the original place and rebuild the equilibrium. Comparing this result with the result in experiment 1.1, we find that the fiscal policy is effective while the monetary policy is ineffective at all under the fixed exchange rate regime. Basically, there is simply a swap between domestic credit and the reserve.

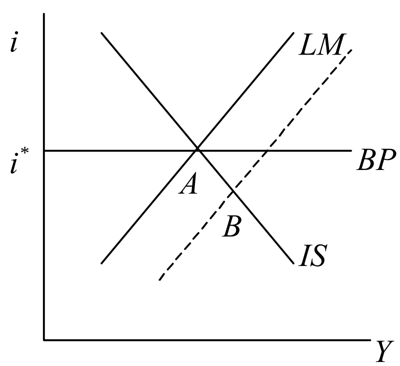

Figure 31

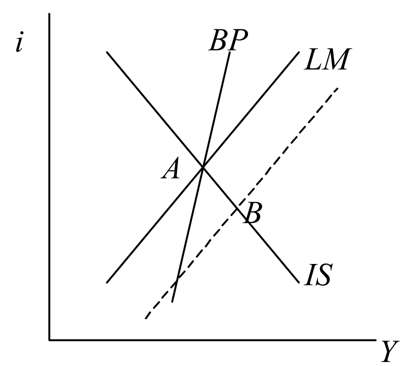

Figure 32

Experiment 1.3 A Temporary Devaluation

The devaluation would shift the IS curve to the right and the BP curve to the right. Depending upon the extent of capital mobility, people would differently react to the changes in interest rate and thus induce different results regarding the BP account. Then the change in reserve level would also help to shift the LM curve so as to bring back the eventual equilibrium.

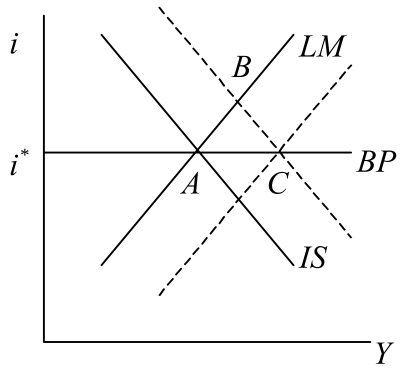

Figure 33

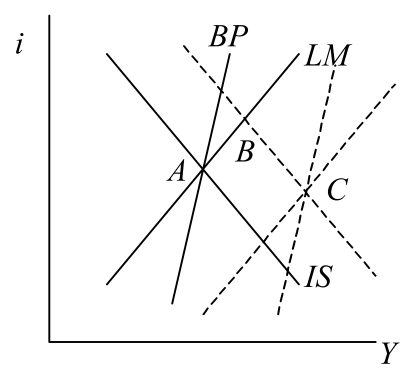

Figure 34

For the case of full capital mobility, the BP curve remains the same and the IS curve shift to the right so as to produce a higher interest rate and output level. The economy temporarily locates at point B. However, since people are extremely sensitive to the change in interest rate, there will be a BP surplus and thus a higher reserve level, which shifts the LM curve to the right until point C. Essentially, both the story and the figures behind the economy are exactly the same as those in experiment 1.1. An important point to note here is that the devaluation would help to increase the reserve level.

For the case of low capital mobility, the implications to the devaluation are kind of complicated. Suppose a 5% increase in output would offset the increase of S in the BP equation. This implies in the IS market there exists excess supply so that the output has to decrease a little bit to shrink the supply while increasing the demand from net export, i.e., the increase in output would be less than 5% in the IS equation. The more shift of BP curve to the right than that of the IS curve therefore bring the economy at point B. Comparing to the new BP curve, there is a surplus at point B due to the devaluation. The higher reserve then shifts the LM curve to the right until point C where three curves intersect again. Therefore, the final equilibrium level of interest rate will be lower and that of output will be higher.

2. Flexible Exchange Rate Regime

Under the flexible exchange rate regime, the exchange rate has to change to maintain the reserve level in the BP equation. (We couldn't simultaneously determine four unknowns, Y, i, S and R, using only three equations.) Since the reserve level is fixed, there will be no impact on the LM schedule from the BP account, and the economy will always stay on the LM curve. The LM curve will not shift unless the domestic credit has been raised. Also note that once the exchange rate depreciates, both the IS curve and BP curve would shift to the right. From experiment 1.3, we have known one way of interpreting why the BP curve would shift more to the right than the IS curve when the home currency depreciation occurred. Alternatively, suppose the IS curve shift to the right a certain amount, given the domestic interest rate. The fact that the output consists of both domestic expenditure and trade surplus implies that the trade surplus must be higher corresponding to the higher output. Then the same amount of shift of BP curve won't reach the balance in BP account due to the trade surplus, and the BP curve has to shift further to the right.

Experiment 2.1 Fiscal Expansion (Increase in Government Spending)

The expansion of government spending shifts the IS curve to the right and the economy temporarily reaches point B (or, point B corresponds to the case of closed economy) with a higher interest rate and output level. Here we are implicitly assuming that the expansion of government spending was all used on domestic goods.

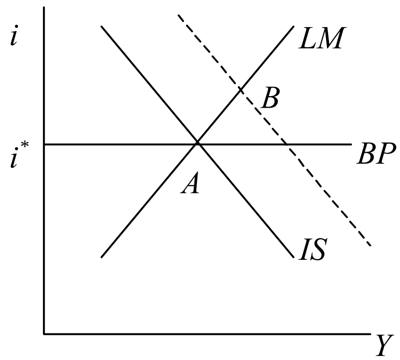

Figure 35

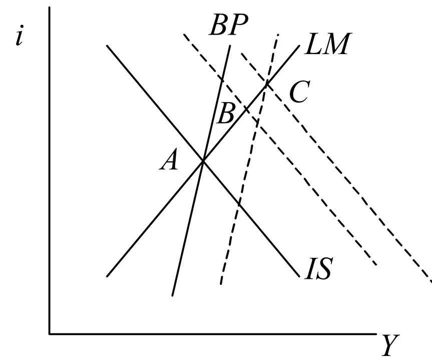

Figure 36

For the case of perfect capital mobility, people are extremely sensitive to the increase of interest rate so that there is a BP surplus and the point B locates above the BP curve. In order to restore BP account balance, the domestic currency has to appreciate, which implies that the IS curve has to shift back to the original place and the economy returns in equilibrium at point A with a nominal exchange rate appreciation. In the case of closed economy, there is certain amount of crowding out effect associated with the fiscal expansion, due to the higher interest rate retarding the higher investment. In the case of small open economy, the fiscal policy is ineffective at all attributing to the crowding out effect from the nominal appreciation that is necessary to bring the BP account back into balance. Basically, there is only a swap between government spending on the domestic goods and the trade balances.

For the case of low capital mobility, people pay more attention to the increase of output level so that there is a BP deficit and the point B locates below the BP curve. In order to restore BP account balance, the home country has to devalue its currency. Then the IS curve has to shift further to the right and the BP curve does the same until point C where all three curves intersect again. In this case the fiscal policy is effective because of the nominal depreciation boosts the economy.

Experiment 2.2 Monetary Policy Expansion (Increase in Domestic Credit)

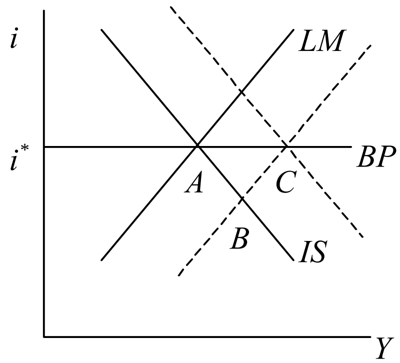

Figure 37

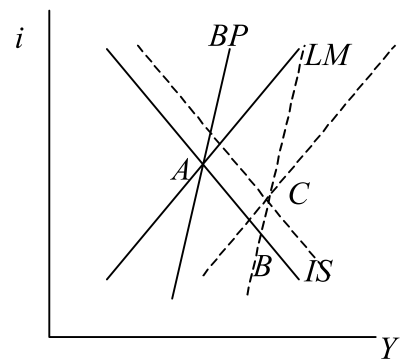

Figure 38

The higher supply of domestic credit implies the LM curve shifts to the right and produces a higher output and lower interest rate level. As we find before, regardless of capital mobility, the BP account will experience a deficit, which implies a devaluation of the domestic currency. The devaluation then shifts both the IS and BP curves to the right. There will be an unambiguous increase of final equilibrium level of output. The final equilibrium level of interest rate for the case of perfect capital mobility will be the same as the original level and that for the case of low capital mobility will be graphically ambiguous. As we have demonstrated in experiment 1.3, the more shift of BP curve to the right than the IS curve therefore determines a lower final equilibrium level of interest rate than before.

3. Mundell-Fleming Model

The Mundell-Fleming model deals with the cases with perfect capital mobility, and yields the results that under fixed exchange rate regime the fiscal policy is extremely effective whereas the monetary policy is not at all, and that under flexible exchange rate regime only monetary policy works.

4. Extension to the Model by Jeffery Sachs

Sachs believes that it is not realistic to assume the stickiness of nominal wage rate. Instead, he assumes the constancy of real wage rate $w - p_c$, where w is the logarithm of nominal wage rate and p_c the logarithm of consumption price level. Furthermore, define the consumption price level as $p_c = \lambda p + (1-\lambda)sp^*$. Let the output be determined by product wage as $Y = f(w-p)$. Under the flexible exchange rate regime, a nominal depreciation implies a higher consumption price level and thus a higher nominal wage rate. With stickiness of domestic price level p, the higher nominal wage rate implies a lower output level, as opposed to a higher one predicted by the Mundell-Fleming model.

03/25/1999

Subject: Two Simple Monetary Models
Reference: Obstfeld, M. and K. Rogoff (1996) *Foundations of International Macroeconomics*, Cambridge: The MIT Press, Chapter 8, p.p. 511-30

Before we go into the monetary models, it is very useful to keep in mind the following three relations. (1) The Fischer parity equation is $(1+r_{t+1}) = (1+i_{t+1})P_t / P_{t+1}$ or $r_{t+1} \approx i_{t+1} - \mathrm{p}_{t+1}$. (2) The uncovered interest parity (UIP) says that if the agent is concerned with only expected returns then the expected returns from anywhere in the world should be the same, i.e., $1+i_{t+1} = (1+i^*_{t+1})[E_t S_{t+1} / S_t]$ or $i_{t+1} \approx i^*_{t+1} + [(E_t S_{t+1} - S_t)/S_t]$ in levels and $i_{t+1} \approx i^*_{t+1} + (E_t s_{t+1} - s_t)$ in logarithms. (3) The covered interest parity (CIP) is $1+i_{t+1} = (1+i^*_{t+1})[F_{t,t+1} / S_t]$ or $i_{t+1} \approx i^*_{t+1} + [(F_{t,t+1} - S_t)/S_t]$ in levels and $i_{t+1} \approx i^*_{t+1} + f_{t,t+1} - s_t$ in logarithms, where $F_{t,t+1}$ is the forward exchange rate for period $t+1$ as of time t.

Model 1: Fischer Parity & Cagan Money Demand

The Fischer parity states $i_{t+1} = r + E_t p_{t+1} - p_t$. Note that i_{t+1} is the interest rate for the period from the beginning of period t to the beginning of period $t+1$, and the prices here are in logarithms. The Cagan money demand is $M^d / P = L^d(i, Y)$. Assuming the constancy of liquidity elasticity of interest rate, $-\mathrm{h}$, and that of output, f, then we have the following in logarithms, $m^d_t - p_t = -\mathrm{h} i_{t+1} + \mathrm{f} y_t$. If we further impose the money market equilibrium condition $m^d_t = m_t$, where m_t is the money supply, then we have

$$m_t + \mathrm{h} r - \mathrm{f} y_t = (1+\mathrm{h}) p_t - \mathrm{h} E_t p_{t+1}.$$

Solving this forward for p_t, while assuming the transversality condition

$$\lim_{T\to\infty} \left(\frac{\mathrm{h}}{1+\mathrm{h}} \right)^T p_T = 0,$$

we have

$$p_t = \left(\frac{1}{1+\mathrm{h}} \right) \sum_{i=0}^{\infty} \left(\frac{\mathrm{h}}{1+\mathrm{h}} \right)^i E_t (m_{t+i} + \mathrm{h} r - \mathrm{f} y_{t+i}).$$

Assume $r = 0$ and $\mathrm{f} = 0$ for simplicity. Perfect foresight implies that $\overline{p} = \overline{m}$. Suppose there is an unanticipated announcement at period t that the money supply will permanently rise from $\overline{m}_A$ to $\overline{m}_B$ starting period T. What change can we expect for the prices?

It is obvious that before period t, the prices are constant at $p_s = \overline{m}_A, (s < t)$, and after period T, the prices will be constant again at $p_s = \overline{m}_B, (s \geq T)$. When $t \leq s < T$, we could derive the prices as following

$$
\begin{aligned}
p_s &= \left[\sum_{i=0}^{T-s-1}[\mathrm{h}/(1+\mathrm{h})]^i \overline{m}_A + \sum_{i=T-s}^{\infty}[\mathrm{h}/(1+\mathrm{h})]^i \overline{m}_B\right]\frac{1}{1+\mathrm{h}} \\
&= \left[\overline{m}_A \frac{1\left[1-[\mathrm{h}/(1+\mathrm{h})]^{T-s}\right]}{1-[\mathrm{h}/(1+\mathrm{h})]} + \overline{m}_B \frac{[\mathrm{h}/(1+\mathrm{h})]^{T-s}}{1-[\mathrm{h}/(1+\mathrm{h})]}\right]\frac{1}{1+\mathrm{h}} \\
&= \overline{m}_A + [\mathrm{h}/(1+\mathrm{h})]^{T-s}(\overline{m}_B - \overline{m}_A).
\end{aligned}
$$

That is, the price at period t would jump immediately and then gradually rise to the level of $\overline{m}_B$ at period T due to rational expectation. This reaction could be represented in the following figure.

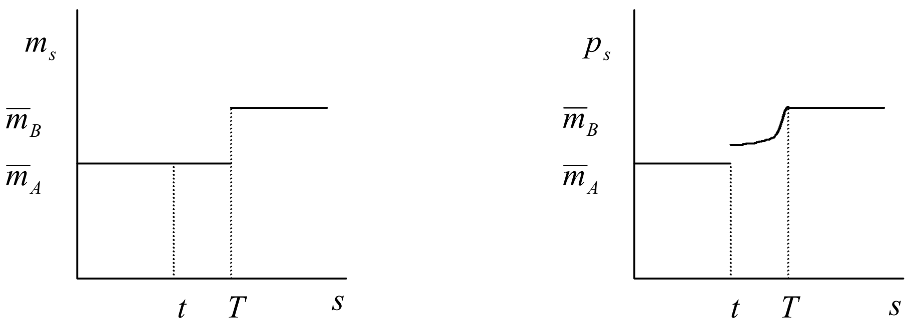

Figure 39

Model 2: Cagan Money Demand & Absolute PPP & UIP

The Cagan money demand once again is $m_t^d - p_t = -\mathrm{h} i_{t+1} + \mathrm{f}\, y_t$, the absolute PPP is $p_t = s_t + p_t^*$, and the uncovered interest parity is $i_{t+1} = i_{t+1}^* + E_t s_{t+1} - s_t$. The money market equilibrium is $m_t = m_t^d$. Doing the similar manipulation as in Model 1, we have

$$m_t + \mathrm{h} i_{t+1}^* - \mathrm{f}\, y_t - p_t^* = (1+\mathrm{h})s_t - \mathrm{h} E_t s_{t+1}.$$

Solving this forward for s_t, and assuming the transversality condition, we have

$$s_t = [1/(1+\mathrm{h})]\sum_{i=0}^{\infty}[\mathrm{h}/(1+\mathrm{h})]^i E_t[m_{t+i} + \mathrm{h} i_{t+i+1}^* - \mathrm{f}\, y_{t+i} + p_{t+i}^*].$$

Suppose $i_{t+i+1}^* = i^*$ and $p_{t+i}^* = p^*$, and there is an expected permanent rise of i^*. From the solution, we know that there will be a nominal depreciation. The intuition behind is the following:

$$i^* \uparrow \Rightarrow i_{t+1} \uparrow \Rightarrow m_t - p_t \downarrow \Rightarrow p_t \uparrow \Rightarrow s_t \uparrow.$$

04/06/1999

Subject: Dornbusch Overshooting Model (I)
Reference: Dornbusch, R. (1976) "Expectations and Exchange Rate Dynamics," *Journal of Political Economy*.
Obstfeld, M. and K. Rogoff (1996) *Foundations of International Macroeconomics*, Cambridge: The MIT Press, Chapter 9, pp. 605-57, Chapter 8, pp. 531-54

The basic setup of the model is the following five equations.

[1] $i_{t+1} = i^* + s_{t+1} - s_t$

[2] $m_t - p_t = -\eta i_{t+1} + \phi y_t$

[3] $y_t^d = \bar{y} + \delta(q_t - \bar{q})$

[4] $q_t \equiv s_t + p^* - p_t$, $\tilde{p}_t \equiv s_t + p^* - \bar{q}$

[5] $p_{t+1} - p_t = \psi(y^d - \bar{y}) + (\tilde{p}_{t+1} - \tilde{p}_t)$

Equation [1] and [2] are uncovered interest parity and Cagan money model. Equation [3] says that the demand for output deviates from the full employment level $\bar{y}$ proportional to the deviation of real exchange rate from the natural rate level $\bar{q}$. $\tilde{p}_t$ is the shadow price level for given s_t while the real exchange rate reaches the natural rate level. Equation [5] delineates the determinants of inflation. Different from the original article, here Obtsfeld and Rogoff add the term $\tilde{p}_{t+1} - \tilde{p}_t$ to allow forward-looking solution.

In this model, we are only concerned with the nominal and real exchange rate so that we need to reduce the system above into a two-equation one. In the goods market side, we do the following manipulation. Combination of equations [3], [4] and [5] implies

$$p_{t+1} - p_t = \psi\delta(q_t - \bar{q}) + (s_{t+1} - s_t) \text{ and } p_{t+1} - p_t = (s_{t+1} - s_t) - (q_{t+1} - q_t).$$

Then we have the following in the goods market side,

[6] $\Delta q_{t+1} = -\psi\delta(q_t - \bar{q})$, where we assume $\psi\delta < 1$.

Let's assume $p^* = \bar{y} = i^* = 0$ and $m_t = \bar{m}$ for simplicity. Then in the asset market side we have $i_{t+1} = s_{t+1} - s_t$, $p_t = s_t - q_t$, $y_t^d = \delta(q_t - \bar{q})$, and thus $\bar{m} - (s_t - q_t) = -\eta(s_{t+1} - s_t) + \phi\delta(q_t - \bar{q})$. Therefore, the equation in the assets market side is

[7] $\Delta s_{t+1} = (1/\eta)s_t - [(1-\phi\delta)/\eta]q_t - (1/\eta)(\phi\delta\bar{q} + \bar{m})$, where we assume $0 < \phi\delta < 1$.

The system of equations [6] and [7] is a standard first-order dynamic system, which we can both graphically (using phase diagram) and algebraically solve out. Let's do the graphic approach at first. From equation [6], we know the $\Delta q = 0$ curve is simply a vertical line in Figure 40. For any point located to the right of this curve, the equation [6] itself tells us that $\Delta q_{t+1} < 0$ so that it will move horizontally to the left. The intuition behind is $q_t > \bar{q} \Rightarrow y_t^d > \bar{y} \Rightarrow p_{t+1} > p_t \Rightarrow q_t < q_{t+1}$. Similarly we know any point to the left of the $\Delta q = 0$ curve tends to horizontally move to the right. Intuitively, the negative sign in front of q_t ensures the goods market side is stable so that any deviation from the $\Delta q = 0$ curve tends to move back.

In the assets market side, equation [7] implies that the $\Delta s = 0$ curve is $s_t = (1-\phi\delta)q_t + (\phi\delta\,\overline{q} + \overline{m})$, which is a positively sloped curve slightly flatter than the 45-degree line. For any point above the $\Delta s = 0$ curve, equation [7] implies $s_{t+1} > s_t$, i.e., the point would tend to vertically move up, further away from the assets equilibrium. The intuition behind is $s_t \uparrow \Rightarrow p_t \uparrow \Rightarrow m_t - p_t \downarrow \Rightarrow i_{t+1} \uparrow \Rightarrow s_{t+1} \uparrow$. Similarly any point below the $\Delta s = 0$ curve would tend to vertically move down, further away from the assets equilibrium. The assets market is an exploding system because of the positive sign in front of s_t in equation [7].

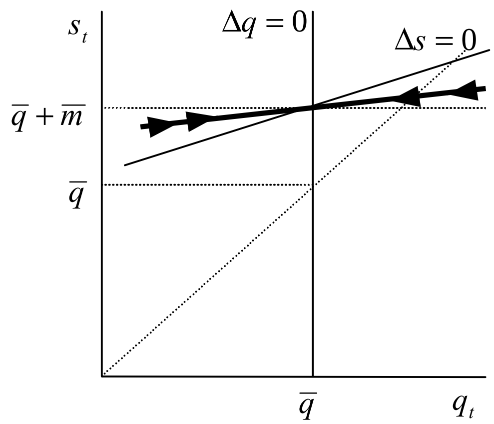

Figure 40

At the intersection point between the $\Delta q = 0$ and $\Delta s = 0$ curves, we have $s_t = \overline{q} + \overline{m}$, $q_t = \overline{q}$, $p_t = \overline{m}$ and $i_t = 0$. The phase diagram determines that there exists a unique stable arm as depicted in Figure 40. For any point on the stable arm that is to the right of the steady state, q_t falls faster than s_t since p_t is rising, i.e., the real exchange rate appreciates more than the nominal one. For any point on the stable arm that is to the left of the steady state, q_t rises faster than s_t since p_t is falling, i.e., the real exchange rate depreciates more than the nominal one.

04/08/1999 & 04/13/1999

Subject: Dornbusch Overshooting Model (II)
Reference: Dornbusch, R. (1976) "Expectations and Exchange Rate Dynamics," *Journal of Political Economy*.
Obstfeld, M. and K. Rogoff (1996) *Foundations of International Macroeconomics*, Cambridge: The MIT Press, Chapter 9, pp. 605-57, Chapter 8, p.p. 531-54.

Application 1: Unexpected Permanent Increase of Money Supply (Effective Immediately)

Suppose at period 0 there is an unexpected permanent increase of money supply from $\overline{m}$ to $\overline{m}'$. It is apparent that in the long run both the price level and the nominal exchange rate will increase correspondingly to the $\overline{m}'$ level and the nominal interest rate will remain at level of zero. However, in the short run, the price level cannot adjust immediately so that $p_0 = \overline{m}$. Equation [4] implies then $s_0 = q_0 + \overline{m}$, which is a straight line through the original steady state point A with slope of one. Since the new steady state is point C and the associated new stable arm SS' is at a higher level than the original one, the economy has to jump to point B at period 0. After that, the economy follows the new stable arm to reach the new steady state in that the price now rises and there will be a nominal depreciation. (If the price still doesn't change, then the economy will return back to original steady state.) From point A to point B, the amount of nominal depreciation is higher than the amount of increase in money supply, and thus this reaction is called overshooting of exchange rate. This is presented in Figure 41.

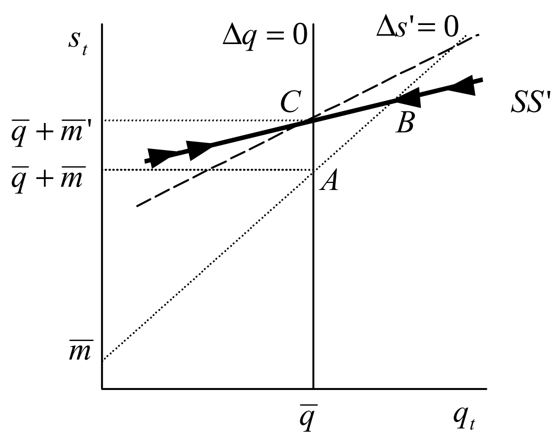

Figure 41

Intuitively, by the assumption of $\phi\delta < 1$ the money demand is not very sensitive to output rather than interest rate. The price stickiness at period 0 implies excess money supply and thus lower interest rate is called for to restore the money market equilibrium. The lower interest rate then implies a nominal appreciation, by the uncovered interest parity. Therefore, at period 0, there has to be a larger amount of depreciation than necessary to reach the new steady state, in order to facilitate the occurring of nominal appreciation.

We can also mathematically derive the results above. First of all, we derive the stable arm from solving the system of two difference-equations [6] and [7].

Equation [6] implies

$$q_{t+1}-\bar{q}=(1-\psi\delta)(q_t-\bar{q}),$$

and thus

$$q_{t+i}-\bar{q}=(1-\psi\delta)^i(q_t-\bar{q}).$$

Equation [7] implies

$$[m_t+(1-\phi\delta)(q_t-\bar{q})]-(s_t-\bar{q})=-\eta(s_{t+1}-s_t),$$

and thus

$$s_t-\bar{q}=\left(\frac{1}{1+\eta}\right)\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i[m_{t+i}+(1-\phi\delta)(q_{t+i}-\bar{q})].$$

Assuming $m_t=\bar{m}$ and plugging the results from equation [6] into the result from equation [7], we have

$$s_t=\bar{q}+\bar{m}+\left(\frac{1-\phi\delta}{1+\psi\delta\eta}\right)(q_t-\bar{q}),$$

and this is the equation that delineates the stable arm.

At point A, $s=\bar{m}+\bar{q}$, $q=\bar{q}$, and $p=\bar{m}$. Combining the new stable arm

$$s_t=\bar{q}+\bar{m}'+\left(\frac{1-\phi\delta}{1+\psi\delta\eta}\right)(q_t-\bar{q})$$

and the 45-degree line

$$s_t=q_t+\bar{m},$$

we have at point B,

$$s=(\bar{m}'+\bar{q})+\left(\frac{1-\phi\delta}{\psi\delta\eta+\phi\delta}\right)(\bar{m}'-\bar{m}).$$

For the case of $\phi\delta<1$, $s>\bar{m}'+\bar{q}$ is supporting the overshooting of nominal exchange rate. Therefore, whether there will be an overshooting is fully depend upon the sensitivity of output with respect to exchange rate and the sensitivity of money demand with respect to output.

<u>Application 2: Anticipated Permanent Increase of Money Supply</u>

Suppose at period n, there is an announcement that the money supply will be permanently increased up to a new level $\bar{m}'$ starting at period N.

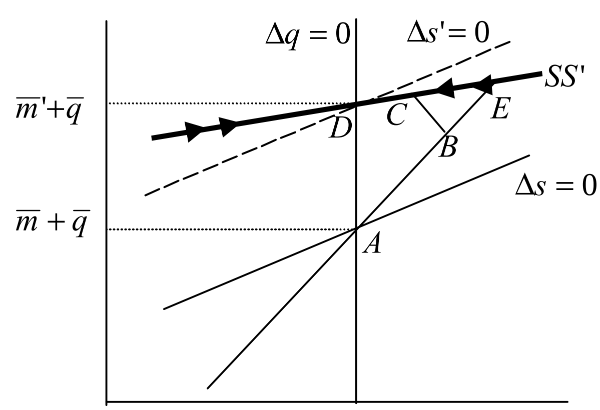

Figure 42

Before the announcement, the economy locates at point A. From the period of announcement to the period of enforcing the new policy, the dynamics of the original system is still in effect, i.e., the directions of motion in the original system are still valid. Once the new policy is effective, the final equilibrium point will be D, associated with the new $\Delta s' = 0$ curve and the new stable arm SS'.

At period n, the price stickiness implies that exchange rates will follow the 45-degree line $s = q + \overline{m}$. Following the directions of motion in the original system, we determine that the economy will jump to a point B somewhere between A and E; for positions elsewhere in the 45-degree line, there is no hope for the economy to reach the new stable arm once the new policy is effective.

During the period n and N, the economy will move in the direction toward the new stable arm, guided by the directions of motion in the original system. At period N, the economy has to reach point C on the new stable arm, then follows the new stable arm until reaching the final equilibrium point D.

It is important to notice that during period from point A to B, there are both nominal and real depreciation. This happens when people expect that some day in the future the growth rate of money supply will rise, despite the fact that nothing about policy has been actually changed. From point B to C, the price level has to rise in the following sense. An unchanged price level will make the economy return back to point A. A falling price will produce a real depreciation by the definition of real exchange rate, given a nominal depreciation, and this is not suggested by the directions of motion in the original system. Given the low sensitivity of money demand with respect to output, the higher price level without the change of money supply produces excess money demand, and thus a higher interest rate is called for to suppress the excess money demand. The higher interest rate then introduces a nominal depreciation from the uncovered interest parity. From point C to D, the higher money supply produces excess money supply and a lower interest rate is called for to bridge the gap between demand for and supply of money, and then the lower interest rate results in a nominal appreciation from the uncovered interest parity.

Essentially in this model, because of the stickiness of price level, something has to change more than otherwise; it turns out nominal exchange rate plays such a role.

04/13/1999 & 04/20/1999 & 04/21/1999

Subject: Exchange Rate and Current Account Model
Reference: Dornbusch, R. and S. Fischer (1980) "Exchange Rates and the Current Account," *American Economic Review*, December

Before we discuss the relationship between exchange rate and current account, it is useful to distinguish two closely related concepts: seignorage and inflation tax. Formally, seignorage is the revenue the government collects by printing money, i.e., $(M_t - M_{t-1})/P_t$, and the inflation tax is the capital loss inflected by inflation on real balances holding, i.e., $[(P_t - P_{t-1})/P_t][M_{t-1}/P_{t-1}]$. Since

$$\frac{P_t - P_{t-1}}{P_t}\frac{M_{t-1}}{P_{t-1}} = \left(1 - \frac{P_{t-1}}{P_t}\right)\frac{M_{t-1}}{P_{t-1}} = \frac{M_{t-1}}{P_{t-1}} - \frac{M_{t-1}}{P_t}, \text{ and } \frac{M_t - M_{t-1}}{P_t} = \left[\frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}}\right] + \left[\frac{M_{t-1}}{P_{t-1}} - \frac{M_{t-1}}{P_t}\right],$$

we have

$$\text{seinorage} = \Delta \text{real balances} + \text{inflation tax}.$$

This is, the government can collect seignorage from the changes in real balances holding, without necessarily creating inflation.

Suppose there is a small open economy with exogenous composite output Y_t for each period. We assume the PPP holds and foreign price is normalized to 1, i.e. $S_t = P_t$. In this economy, there are two forms of assets, one is the real balance holding, and the other is the real foreign bond F_t that bears constant real interest rate r^*. Here foreign bonds and domestic bonds are assumed to be perfectly substitutable so that only foreign bonds are considered. One of the benefit from considering only foreign bonds is that the current account is exactly the change of foreign bonds, according to $CA = \Delta NFA$. There are also government spending G_t and transfer T_t.

In the consumer side, consumption is determined as

[1] $C_t = C(Y_t^d, V_t)$, where $0 < C_{Y^d} < 1, C_{V_t} > 0$.

The real disposable income Y_t^d and the wealth level V_t are defined as

[2] $Y_t^d = Y_t + T_t + r^* F_t - \pi_t (M_t / P_t)$,

[3] $V_t = [(M_t / P_t) + F_t] + [\tilde{Y}_t + \tilde{T}_t]$.

Note that the inflation tax part in the disposable income is written as the product of current inflation and current real balances, not past real balances as required by definition. This is for simplicity purposes. The wealth level consists of current asset holding and present value of future incomes, where $\tilde{Y}_t + \tilde{T}_t = \int_{s=t}^{\infty} (Y_s + T_s) e^{-r^*(s-t)} ds$. Technically we should include the interest revenue from last period bonds holding as well, but this won't change the issues we are going to discuss soon. Also note that by definition of inflation, we have $\pi_t \equiv \hat{P}_t = \hat{S}_t$.

We further assume that people would like to hold only a portion of their wealth in the form of real balance, and the share $L(r^* + \pi_t)$ depends upon the opportunity cost of holding money. Denote the desired level of real balance as l_t, we have $l_t = L(r^* + \pi_t)V_t$. The condition for money market equilibrium is

[4] $M_t / P_t = l_t = L(r^* + \pi_t)V_t$.

This will also imply that

[4’] $\hat{M}_t - \hat{P}_t = \dot{l}_t / l_t$ or $\mu_t - \pi_t = \dot{l}_t / l_t$, if $\mu_t \equiv \dot{M}_t / M_t \equiv \hat{M}_t$.

From this relationship, it is clear that given $\mu_t = 0$, an increase in real balance holding implies a deflation and thus a nominal appreciation.

We close the model by considering the government flow budget constraint. For each period, the government spending and transfer have to be financed through the collection of seignorage; that is,

[5] $T_t + G_t = \dot{M}_t / P_t$.

Since $\dot{M}_t / P_t = (\dot{M}_t / M_t) \cdot (M_t / P_t) = \mu_t l_t$, we also have

[5’] $G_t = \mu_t l_t - T_t$.

The two variables we are most concerned with in this economy are the real balance holding and foreign assets holding. Because of the relationship that a higher real balance holding, given zero money growth rate, implies a nominal appreciation, these two variables are also about nominal exchange rate and current account.

From equation [4], we have

$$r^* + \mu_t - \dot{l}_t / l_t = L^{-1}(l_t / V_t).$$

Let’s define $L^{-1}(l_t / V_t) \equiv \phi(l_t / V_t) = \phi[l_t /(F_t + \tilde{Y}_t + \tilde{T}_t)]$. The second equal sign comes from $l_t / V_t \uparrow \Leftrightarrow [l_t /(F_t + \tilde{Y}_t + \tilde{T}_t)] \uparrow$. From the relationship above, we know $\phi'(\cdot) < 0$ in that a higher share of real balance holding must reflect the fact that the opportunity cost of holding money must have fallen, i.e., $r^* + \pi_t$ must be lower. This relationship also induce the following dynamic equation

[6] $\dot{l}_t = [r^* + \mu_t - \phi(l_t / V_t)] l_t$.

From $CA = \Delta NFA$, we have $CA_t = \dot{F}_t$. From $CA = \text{national income} - \text{national absorption}$, we have $CA_t = Y_t + r^* F_t - C(Y_t^d, V_t) - G_t$. Therefore, the dynamic equation regarding foreign assets holding is the following

[7] $\dot{F}_t = Y_t + r^* F_t - C(Y_t^d, V_t) + T_t - \mu_t l_t$.

Note that we can also achieve this dynamic equation from the perspective of how consumers spend their disposable income, i.e., $Y_t^d - C_t(Y_t^d, V_t) = \dot{l}_t + \dot{F}_t$.

Suppose there exists a unique equilibrium to the system of equation [6] and [7]. Assume $Y_t = \bar{Y}$, $\mu_t = \bar{\mu}$, and $T_t = \bar{T}$. There are only two policy variables in this economy, $\bar{\mu}$ and $\bar{T}$. To tackle the dynamic issues of this system, we need to find out the directions of motion by looking at $\dot{l} = 0$ and $\dot{F} = 0$ equations.

Firstly, let $\dot{l} = \alpha(l, F) = 0$. For given F and $\bar{\mu} = 0$, a higher real balance holding level l_t implies a higher money holding share, by $\phi[l_t /(F_t + \tilde{Y}_t + \tilde{T}_t)]$. This higher money holding share can be achieved only if the opportunity cost of holding money has fallen, and then $\mu - \pi_t = \dot{l}_t / l_t$ implies an even higher real balance holding level. For given l and $\bar{\mu} = 0$, a higher bonds holding level F_t implies a lower money holding share, by $\phi[l_t/(F_t + \tilde{Y}_t + \tilde{T}_t)]$, and thus we will have a lower real balance holding level due to the same logic. That is, we can write $\dot{l} = \alpha(l, F) = \alpha(+,-) = 0$.

Mathematically, $\dot{l} = 0$ implies $r^* + \bar{\mu} = \phi[l_t/(F_t + \tilde{Y}_t + \tilde{T}_t)]$ or $l_t = 0$. Using implicit function theorem, we have $\alpha_l = -\phi'(\cdot)L(1-L) > 0$ and $\alpha_F = \phi'(\cdot)L^2(1-L)^2 < 0$, where $L = l_t / V_t$. It is obvious that the money holding share is constant so that the $\dot{l} = 0$ curve is a positive sloping straight line through the origin, as in Figure 43.

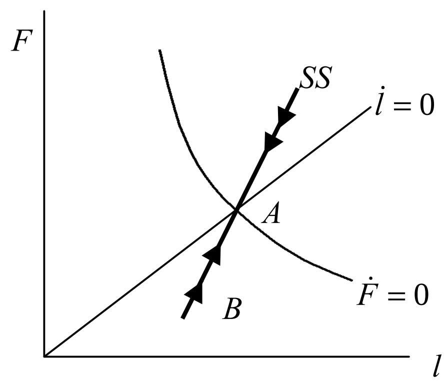

Figure 43

Secondly, let $\dot{F} = \beta(l, F) = 0$. For given F and $\bar{\mu} = 0$, a higher real balance holding l_t increases the current wealth level. Nothing happened to the national income, but the higher wealth level implies a higher consumption level. The higher real balance holding also implies higher seignorage revenue so that government spending will rise for any given level of transfer. Therefore, the impact of a higher real balance holding on current account is unambiguously negative. For given l and $\bar{\mu} = 0$, a higher foreign bonds holding increases both the national income and the disposable income and thus the consumption level. The net impact of a higher foreign bonds holding level on the current account will depend upon which of these two effects are bigger. That is, we can write $\dot{F} = \beta(l, F) = \beta(-,?) = 0$.

Mathematically, we can derive from $\dot{F} = 0$, using implicit function theorem, that

$$\beta_l = -\bar{\mu}(1 - C_{Y^d}) - C_V + [C_{Y^d}\phi'(\cdot)L/(1-L)] < 0 \text{ and}$$

$$\beta_F = r^*(1 - C_{Y^d}) - C_V - [C_{Y^d}\phi'(\cdot)L^2/(1-L)^2] ?\, 0.$$

Here we assume $\beta_F > 0$, i.e., the change of foreign bonds holding has a bigger impact on national absorption than on national income. Furthermore, we assume the determinant of the matrix $\begin{pmatrix} \alpha_l & \alpha_F \\ \beta_l & \beta_F \end{pmatrix}$ is negative and the system has one positive eigenvalue and one negative eigenvalue so that there exists a unique equilibrium.

Following the assumption $\beta_F > 0$, we know the curve $\dot{F} = 0$ is downward sloping in Figure 43. And the $\dot{l} = 0$ curve is exploding and the $\dot{F} = 0$ curve is stable. Correspondingly, the stable arm

SS is depicted in Figure 43. For any point B below the steady state A, the adjustment along the stable arm implies a current account surplus for sure. When $\bar{m} = 0$, this adjustment also implies a nominal appreciation; however, when $\bar{m} > 0$, the higher real balance holding over time implies $\bar{m} - p_t > 0$, and the fact that eventually $p = \bar{m} > 0$ implies that there is a nominal depreciation during the adjustment process along the stable arm, despite the speed of depreciation is shrinking over time.

<u>Application 1: Unanticipated permanent increase in money supply level (effective immediately)</u>

Initially the economy stays at the steady state A and $m_A = 0$. Suppose there is an unexpected increase in money supply from M_A to M_B, but no change in money growth rate. Apparently there is nothing exogenous changed so that the dynamic system won't change, and the economy will stay at the original place, point A. Neither current account nor nominal exchange rate changed, and only price level jumped immediately proportionally to the increase of money supply level.

<u>Application 2: Anticipated permanent increase in money supply level (effective in the future)</u>

At period $t = n$, the economy stays at the steady state A, with m_A and $m_A = 0$. People are told that at period $t = N$, the money supply level will be permanently increased to $m_B > m_A$, while the growth rate of money supply remains zero, i.e., $m_B = 0$. From the results in application 1, it is clear that there is nothing exogenous happened so that the whole dynamic system will remain the same as before; the economy will eventually return back to point A.

At period $t = n$, the news comes along and the foreign bonds holding cannot be changed. In response to the expectation of a higher money supply in the near future, the price level starts to jumps immediately so that the real balance holding level reduces. The economy jumps immediately to point B. During the period $t = n$ and $t = N$, the economy evolves along the forces under the dynamic system and reaches point C. Once the change in the monetary policy is effective, the real balance holding jump immediately from point C to point D on the stable arm. From then on, the economy follows the stable arm to return back to the original equilibrium point A. Figure 44 designates such an evolvement of the economy.

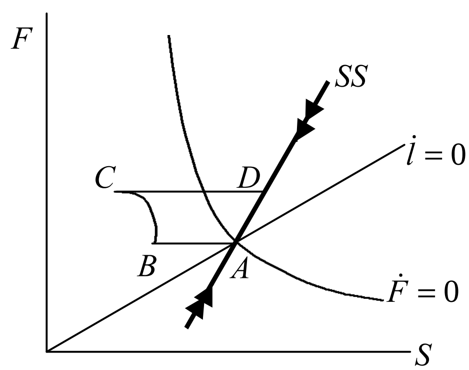

Figure 44

<u>Application 3: Unanticipated permanent increase in money supply growth (effective immediately)</u>

Initially the economy stays at the steady state A and $\mu_A = 0$. Suppose there is an unanticipated permanent increase in money supply growth rate up to $\mu_B > \mu_A$ and this policy change is effective immediately. Since $\pi_t = \mu_t - \dot{l}_t / l_t$, the higher money supply growth rate implies a higher opportunity cost of holding money, and thus the money holding share $L = l_t / V_t$ will shrink. For given F_t level, this implies a lower real balance holding level. Therefore, the $\dot{l} = 0$ curve will rotate to the left, corresponding to the policy change. For given l_t level, the higher money supply growth rate won't change the national income, but will increase the seignorage revenue so as to have a higher government spending given the transfer. Therefore, the $\dot{F} = 0$ curve will shift to the left. It is clear that the new steady state level real balance holding will fall unambiguously, but the net impact on the foreign bonds holding at the new steady state is not clear.

Case 1: Suppose the $\dot{F} = 0$ curve shifts to the left less than the rotation of $\dot{l} = 0$ curve. The new stable arm associated with the new dynamic system will be SS' in Figure 45. It is clear that at the moment when agents know the unexpected policy change, the holding of foreign bonds won't change. Therefore, the real balances holding has to fall immediately from the original steady state A to point B on the new stable arm, and the economy will then follow the new stable arm to reach the final equilibrium C.

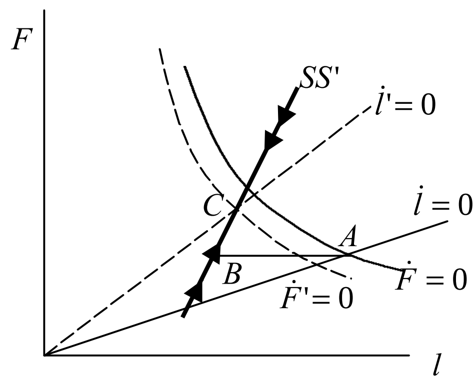

Figure 45

From point A to B, there is no change in current account due to stickiness of foreign bonds holding, and the higher jump of price than the jump of money supply induces a lower real balance holding. During this period, the current account remains unchanged and there is a nominal depreciation. From point B to C, there is a current account surplus and a nominal depreciation. Note that the bounding back of real balance holding implies $\mu_B - \pi_t > 0$, and that $\pi_t > 0$ must hold because eventually at the new steady state C one has $\pi = \mu_B > 0$. That is, prices are rising and the growth rate of π_t is also rising over time from B to C, i.e., there is a nominal depreciation.

Also note that there is an overshooting of real balance holding due to the stickiness of foreign bonds holding. Comparing the two steady states, we find there is a big portfolio reallocation in that l falls and F rises so as to have a lower money holding share $l / (F + \tilde{Y} + \tilde{T})$.

We see a large rotation in $\dot{l} = 0$ curve in this case, which means that for given F, we need to reduce a large amount of real balance holding to reach the new $\dot{l}' = 0$ curve associated with the higher money supply growth. The only reason for this is that people are extremely sensitive to the opportunity cost of holding money so that a small change in money growth rate, which increases the opportunity cost of holding money, will induce a big portfolio reallocation. We also see a relatively small shift in $\dot{F}=0$ curve in this case, which means that for given l, we need to reduce a small amount of foreign bonds holding to reach the new $\dot{F}'=0$ curve associated with the higher money supply growth. As we have noted above, the higher money supply growth rate implies higher seignorage revenue and thus higher government spending, and thus there must be some kind of shrinking effect on the national income net private consumption. By our assumption, the impact of change in F on national income is stronger than that on national absorption. Hence it must be the case that private consumption is very sensitive to wealth changes so that only a small amount of reduction of F is needed to cause a strong enough reduction of private consumption that is larger than the reduction of national income.

Case 2: Suppose the $\dot{F}=0$ curve shifts to the left more than the rotation of $\dot{l}=0$ curve. The new stable arm associated with the new dynamic system will be SS' in Figure 46. Once again we have a sticky foreign bonds holding and the real balance holding has to fall immediately from the original steady state A to point B on the new stable arm, and the economy will then follow the new stable arm to reach the final equilibrium C.

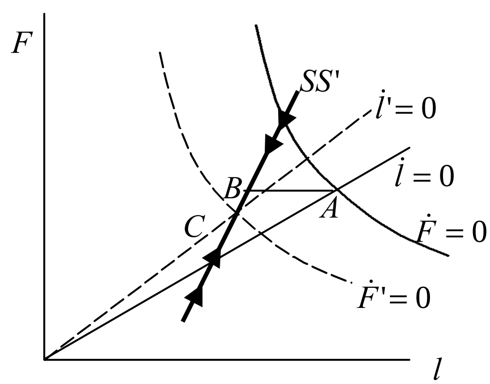

Figure 46

In this case, assets demand is not very sensitive to returns so that the $\dot{l}=0$ curve is rotating less than the left shifting of the $\dot{F}=0$ curve, which stands for the fact that private consumption is not sensitive to the wealth changes, either. Because of this, there is no big reallocation of assets comparing the two steady states. From A to B, the current account remains the same as before and there is a nominal depreciation. From B to C, the economy experiences a current account deficit and nominal depreciation whose speed is shrinking over time.

<u>Application 4: Anticipated permanent increase in money supply growth (effective in the future)</u>

Very similar to application 3, there are two possible cases to consider. Here we consider only the case where assets demand is very sensitive to returns and private consumption is also very sensitive to the wealth changes. Suppose the economy originally locates at point A in Figure 47 with $\mu=0$. At time $t=n$, there is an announcement that by time $t=N$, there will be a permanent increase of money supply growth up to $\mu'>0$, effective immediately at period $t=N$.

It is clear that after the new policy is enforced the economy will locate at the new steady state D. But before $t = N$, the dynamics of the economy will be still governed by the old system. Having the directions of motion under the old system in mind, we know that immediately after the announcement, the holding of foreign bonds cannot be changed, and only price could change. Suppose the price doesn't change at time $t = n$, then the old system will make sure the economy stays at point A until time $t = N$. However, this will not be able to bring the economy to the new stable arm once the new policy is enforced. Similarly, the jump down of price level won't bring the economy to the new steady state, either. The economy has to jump to a point B to the left of A, standing for a lower real balance holding level because of the inflation. Then the economy will move in the northwest direction following the motions in the old system until at time $t = N$ it reaches point C on the new stable arm. The economy will eventually follow the new stable arm to reach the new steady state D after that.

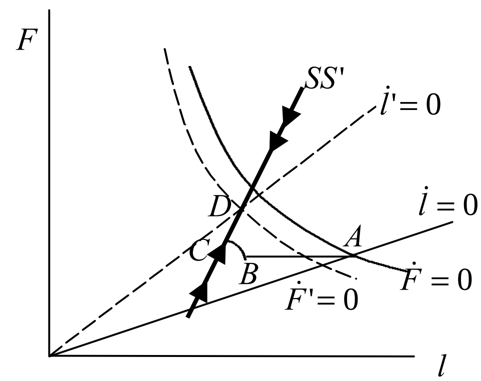

Figure 47

Clearly, from point A to B, the current account remains the same as before and there is a nominal depreciation. From B to C, there is a current account surplus and further nominal depreciation. From C to D, there is a current account surplus and further nominal depreciation. Once again, there is an overshooting of real balance holding and a big reallocation of portfolio because of our assumption that asset demand is very sensitive to returns and the private consumption is also very sensitive to the wealth changes.

04/21/1999 & 04/22/1999

Subject: First Generation Currency Crisis Model
Reference: Flood, R. and N. Marion (1997) "Perspectives on the Recent Currency Crisis Literature," *Dartmouth College Working Paper*, September.
Krugman, R. (1979) "A Model of Balance of Payment Crises," *Journal of Money, Credit, and Banking*, August.
Flood, R. and P. M. Garber (1984) "Collapsing Exchange Rate Regimes: Some Linear Examples," *Journal of International Economics*, August.

Let Mexico be the small economy under consideration. The nominal exchange rate is defined as $S = peso/\$$. Assume that agents have perfect foresight, and the money market equilibrium condition is

[1] $m_t - p_t = -a i_t$.

In levels, we have the accounting identity $M = DC + R$. A linear approximation (not very precise but simplify the algebra a lot without changing the essential taste of the model) is

[2] $m_t = d_t + r_t$

in logs, where d_t is the domestic credit and r_t foreign reserve. The purchasing power parity is assumed to hold,

[3] $p_t = s_t + p_t^*$.

And the uncovered interest parity also holds,

[4] $i_t = i^* + (s_{t+1} - s_t)$.

For simplicity, we assume $i^* = p_t^* = 0$. We also assume that the government has to collect seignorage revenue to finance fiscal needs, and in particular, the domestic credit expands at a constant rate m. That is,

[5] $\dot{d}_t = m$,

where the dot stands for differential and thus the growth rate of the corresponding variable in level.

1. Credible Fixed Regime

Suppose the central bank has an open window to support the designated fixed exchange rate $s_t = \bar{s}$ so that $s_{t+1} - s_t = 0$. Further assume that the economy has enough reserve to maintain such a regime without speculator's attack. The combination of the equations above yields

$$d_t + r_t - s_t = -a i_t = 0\text{, and thus } \dot{d}_t = -\dot{r}_t.$$

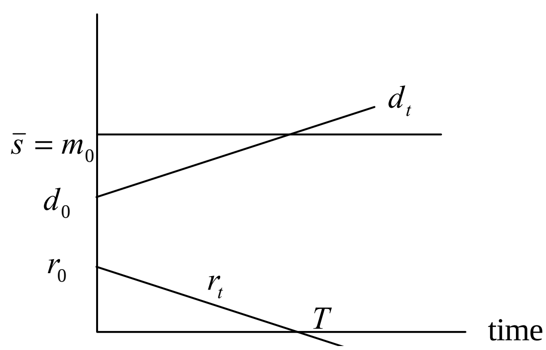

Figure 48

Given initial condition d_0 and r_0, we have

$$d_t = d_0 + \mu t,\ r_t = r_0 - \mu t \text{ and } \bar{s} = m_0 = d_0 + r_0 .$$

They are represented in the Figure 48.

It is apparent that by period T such that $r_T = 0 \Leftrightarrow r_0 - \mu T = 0 \Leftrightarrow T = r_0 / \mu$, the reserve will be exhausted even if there is no speculator's attack. Once the central bank's reserve is exhausted, there are basically two options, one is to devalue and the other is to switch the regime into a flexible one. Here we assume that the central bank will change the regime if $r_t = 0$. Then it is not optimal for the speculators to attack right at period T, and they tend to act at least one period ahead, which will infinitely push others to act one further period ahead. Therefore, this idea that the reserve will be exhausted by period T is not valid.

2. Shadow Exchange Rate

Here we consider a more realistic version of timing. Suppose the central bank decides to switch the regime into flexible one once the reserve is exhausted. Denote by $\tilde{s}_t$ the shadow exchange rate given the float regime and $r_t = 0$. Then the system above reduces to

$$d_t - \tilde{s}_t = -\alpha \dot{\tilde{s}}_t .$$

From this equation, we cannot say there is no steady state growth rate for $\tilde{s}_t$ because d_t is growth over time. Let us differentiate it one more time and get

$$\dot{d}_t - \dot{\tilde{s}}_t = -\alpha \ddot{\tilde{s}}_t \Leftrightarrow \mu - \dot{\tilde{s}}_t = -\alpha \ddot{\tilde{s}}_t .$$

Now we are sure that there is no steady state growth rate for $\dot{\tilde{s}}_t$, but steady state levels for $\dot{\tilde{s}}_t$ at μ, i.e., the steady state growth for $\tilde{s}_t$. Equivalently, there is a constant nominal interest rate at $i_t = \dot{\tilde{s}}_t = \mu$. Then we have

$$d_t - \tilde{s}_t = -\alpha\mu \Leftrightarrow \tilde{s}_t = d_t + \alpha\mu,$$

which is represented in Figure 49.

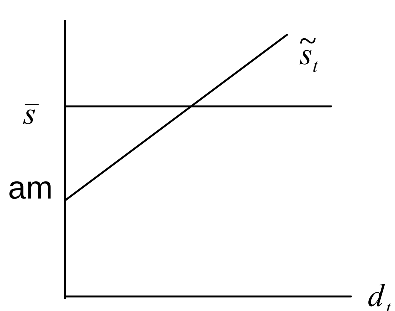

Figure 49

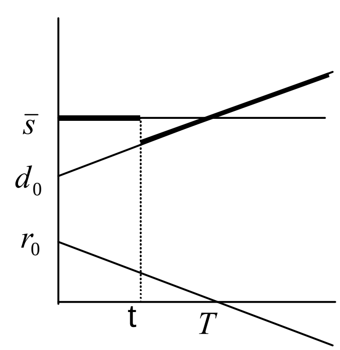

Figure 50

Define τ such that

$$\tilde{s}_\tau = \bar{s} \Leftrightarrow d_0 + \mu\tau + \alpha\mu = \bar{s}(= d_0 + r_0) \Leftrightarrow \tau = (r_0 - \alpha\mu)/\mu = T - \alpha .$$

Before the time period τ, any attempt to attack the central bank's reserve would end up with a capital loss. Any time after the time period τ, any attempt to attack the central bank's reserve would end up with a capital gain, which means the speculators should attack infinitely small period ahead. The final result will be that the optimal time to attack is at period t .

We know the optimal attacking time t is earlier than the time T under the credible fixed regime. Note that t depends upon r_0, m and a . A higher initial reserve level r_0 will prolong the time span until being attacked, a higher growth rate of domestic credit expanding will shrink the time span until being attacked, and a higher money demand elasticity to interest rate will shrink the time span until being attacked. Also note that money supply fixed at the $\bar{s}$ level until period t , drop to the level of d_t at period t , and then follows the path of d_t after period t . The darkened line segment in Figure 50 represents the path of money supply.

Suppose the central bank is trying to sterilize the reserve by using open market operation, then at time T, we have $\tilde{s}_T > \bar{s}$, which means the speculators should have attacked the central bank earlier than T. Therefore, the eventual result is that once the central bank announces the attempt to sterilize the reserve, the speculators are going to attack.

The model above is the so-called first generation currency crisis model, which emphasizes the fundamentals. However, there are many flaws with this model. One of them is that this model predicts that when the currency crisis occurs there is a drop of money supply, which is often not the case empirically. Flood and Marion (1997) provides a fix to this model by revising the uncovered interest parity equation so as to include a component of risk premium on bonds holding. The risk premium on bonds holding will change once the central bank announces the attempt to sterilize the reserve so that the regime won't collapse.

04/22/1999 & 04/27/1999

Subject: Second-Generation Currency Crisis Model
Reference: Obstfeld, M. (1994) "The Logic of Currency Crisis," *NBER Working Paper*, #4640.

While first-generation models emphasize the fundamentals, the second-generation models deal with the phenomena of many crises without obvious deterioration of fundamentals. One approach is to add non-linearity into the model through the government spending. Suppose at the fixed regime, the domestic credit won't expand, i.e., $m = 0$; however, when the economy is switched into floating one, the domestic credit follows $\dot{d}_t = \dot{m} > 0$. If we represent such a system into Figure 51, similar to Figure 48, it is easy to find that in region $d_t \in (d_0, d_B)$ there is no attack and the economy stays at fixed regime, in region $d_t \in (d_A, \infty)$ there is attack for sure, and the region $d_t \in (d_B, d_A)$ is called vulnerable region.

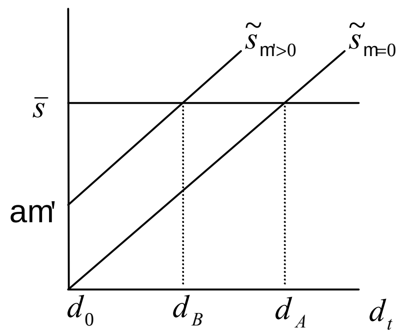

Figure 51

There are many questions to be asked with this model as well. The central bank may try to hide the information of $\dot{m}$ from the public so that people are not so certain of the location of d_B, but once the public is smart enough to know the distribution of $\dot{m}$.

Another approach of the second-generation models is the so-called "escape clause." This was discussed mainly in Obstfeld (1994).

04/27/1999 & 04/29/1999

Subject: Herding Models of Currency Crises
Reference: Chari, V. V. and Patrick Kehoe (1999) "Herds of Hot Money," *Federal Reserve Bank of Minneapolis Research Department*, March

There are $N+1$ investors in the economy where two options of investment exist; one is the safe asset with return R^*, and the other is the risky asset with return R when the state $y \in \{G, B\}$ happens to be good $y = G$, and return 0 when the state y happen to be bad $y = B$. The state path $\{y_t\}$ is unknown but its distribution is common knowledge with the unconditional probability of being in a good state at level μ. The investment decision x_t is the share of investment on risky asset. The public information h_t is the action history $\{x_{t-1}, x_{t-2}, ..., x_0\}$. The private information for each investor consists of both the public information h_t and a private signal $s_t \in \{G, B\}$. The signals are both informative and symmetric such that

$$\Pr(s = G \mid y = G) = \Pr(s = B \mid y = B) = q > \tfrac{1}{2}.$$

Suppose the investor's belief that the economy is at a good state is p, i.e., $\Pr(y = G) = p$. Then the investor's optimization problem is to find out

$$x(p) = \arg\max_x \{x \cdot p \cdot R + x \cdot (1-p) \cdot 0 + (1-x) \cdot R^*\}.$$

Apparently the solution is

$$x(p) = \begin{cases} 1 \text{ if } p \ge R^* / R \\ 0 \quad \text{otherwise} \end{cases}.$$

Investors are only allowed to make sequential decisions in this game. Denote the investor's strategy space as $x_t(h_t, s_t)$, and his belief that the economy is at a good state $p_t(h_t, s_t)$. Then a **perfect Bayesian equilibrium** is $\{x_t, p_t\}_{t=0}^N$ such that

(1) given $p_t(x_t, p_t)$, $x_t(h_t, s_t) = \arg\max_x \{x_t p_t R + (1 - x_t) R^*\}$;

(2) all $p_t(x_t, p_t)$ satisfies Bayes' rule whenever possible.

In order to find out the equilibria, we should be able to find out the beliefs updating rules. Denote as $p_t(h_t)$ the belief before the revelation of the signal, given the public information h_t, and denote the belief after the revelation of the signal s_t as $p_t(h_t, s_t)$. Following the Bayes rule, we obtain

$$p_t(h_t, G) = \frac{p_t(h_t) q}{p_t(h_t) q + [1 - p_t(h_t)](1-q)} = \frac{q}{q + (1-q)\{[1/p_t(h_t)] - 1\}}, \text{ and}$$

$$p_t(h_t, B) = \frac{p_t(h_t)(1-q)}{p_t(h_t)(1-q) + [1 - p_t(h_t)] q} = \frac{(1-q)}{(1-q) + q\{[1/p_t(h_t)] - 1\}}.$$

Suppose at the moment that all the signals are also public information. We would like to find out the relationship between $p_t(h_{t+1}, s_{t+1})$ and $p_t(h_t)$. Suppose the state path for two periods is $s_t = G$ and $s_{t+1} = G$. Applying $p_{t+1}(h_{t+1}) = p_t(h_t, G)$ to the updating rule of $p_{t+1}(h_{t+1}, G)$, we have

$$p_{t+1}(h_{t+1}, s_t = G, s_{t+1} = G) = \frac{q^2}{q^2 + (1-q)^2\{[1/p_t(h_t)] - 1\}}.$$

Similarly for other combination of signals for two consecutive periods, we have

$$p_{t+1}(h_{t+1}, s_t = G, s_{t+1} = B) = p_t(h_t),$$

$$p_{t+1}(h_{t+1}, s_t = B, s_{t+1} = G) = p_t(h_t),$$

$$p_{t+1}(h_{t+1}, s_t = B, s_{t+1} = B) = \frac{(1-q)^2}{(1-q)^2 + q^2\{[1/p_t(h_t)] - 1\}}.$$

Given the signal path $s^{t+i} = \{s_{t+i}, s_{t+i-1}, \ldots s_t\}$, let k be the number of good signals minus the number of bad signals, i.e., $k_t(s^{t+i}) = \# G - \# B$. It is easy to derive that the updated beliefs follow

$$p_{t+i}[h_{t+i}, k_{t+i}(s^{t+i})] \equiv P(k) = \begin{cases} \dfrac{q^k}{q^k + (1-q)^k\{[1/p_t(h_t)] - 1\}} \text{ if } k > 0 \\ \dfrac{(1-q)^k}{(1-q)^k + q^k\{[1/p_t(h_t)] - 1\}} \text{ if } k < 0 \\ \\ p_t(h_t) \text{ if } k = 0 \end{cases}.$$

Following the assumption of $q > \frac{1}{2}$, it is easy to prove that $dP(k)/dk > 0, \forall k > 0$ and $dP(k)/dk < 0, \forall k < 0$. A simple application of this rule is $P(1) > P(0) > P(-1)$. Suppose at the initial period $p_0(h_0 = \emptyset) = \mu$, we have $P(0) = \mu$ and $P(-1) = [\mu(1-q)]/[\mu(1-q) + (1-\mu)q]$. We make another assumption about parameters as follows, $\mu > R^* / R > [\mu(1-q)]/[\mu(1-q) + (1-\mu)q]$. This essentially is saying that the investment decision rules are $x(p) = 1$ if $p \geq P(0)$ and $x(p) = 0$ if $p \leq P(-1)$.

Since the signals are actually private information, each investor can only observe the investment decision history $h_t = \{x_{t-1}, x_{t-2}, \ldots x_0\}$, not the signal path $s^{t-1} = \{s_{t-1}, s_{t-2}, \ldots s_0\}$, he has to infer the signal received by each investor from their investment decisions. Therefore, we also need to design an inferring rule as follows.

(1) On the equilibrium path, if all the investors make decision consistent with the signal received, then the next investor infer the signal path from the investment decision, i.e., $s_t^? = G$ for $\{x_t(p_t(h_t, G)) = x_t(p_t(h_t, B)) = 1\}$ and $s_t^? = B$ *for* $\{x_t(p_t(h_t, G)) = x_t(p_t(h_t, B)) = 0\}$.

(2) Off the equilibrium path, if all the investors make decision inconsistent with the signal received, then the next investor cannot infer the signal path from the investment decision, i.e., $s_t^? = ?$ for $\{x_t(p_t(h_t, G)) = 0$, and $x_t(p_t(h_t, B)) = 1\}$.

It can be shown that the strategy space and beliefs specified above, together with the parameter specification, constitute a **perfect Bayesian equilibrium**. We provide an example of the equilibria as follows.

Given $x_0 = 1$ and $h_0 = \emptyset$, the investor 1 infer that $s_0^? = G$ and thus $k_1(s^1) = 1$. Then he forms his belief $p_1(h_1) = P(1)$. Based upon the private signal he receives, he revises his belief as

$p_1(h_1, G) = P(2)$ or $p_1(h_1, B) = P(0)$. His strategy space is $\{x_1(p_1(h_1, G)) = x_1(p_1(h_1, B)) = 1\}$, i.e., he invests regardless of his private signal. Since this is on the equilibrium path, the investor 2 would infer that $s_0^? = G, s_1^? = G$, based upon $h_2 = \{x_1 = 1, x_0 = 1\}$. Once again the investor 2 will also invest regardless of his private signal. Therefore, starting from investor 1 there will be a cascade with investment.

Given $x_0 = 0$ and $h_0 = \emptyset$, the investor 1 infer that $s_0^? = B$ and thus $k_1(s^1) = -1$. Then he forms his belief $p_1(h_1) = P(-1)$. Based upon the private signal he receives, he revises his belief as $p_1(h_1, G) = P(0)$ or $p_1(h_1, B) = P(-2)$. His strategy space is $\{x_1(p_1(h_1, G)) = 1, x_1(p_1(h_1, B)) = 0\}$. Since this is also on the equilibrium path, the investor 2 would infer $s_0^? = B, s_1^? = G$ if the public history is $h_2 = \{x_1 = 1, x_0 = 0\}$, or $s_0^? = B, s_1^? = B$ if the public history is $h_2 = \{x_1 = 0, x_0 = 0\}$. The strategy space for the investor 2 would be:

if $h_2 = \{x_1 = 1, x_0 = 0\}$, then $\{x_2(p_2, G) = 1, x_2(p_2, B) = 0\}$;

if $h_2 = \{x_1 = 0, x_0 = 0\}$, then $\{x_2(p_2, G) = x_2(p_2, B) = 0\}$.

Therefore, we have the public history $h_3 = \{x_2 = 1, x_1 = 1, x_0 = 0\}$, or $h_3 = \{x_2 = 0, x_1 = 1, x_0 = 0\}$, or $h_3 = \{x_2 = 0, x_1 = 0, x_0 = 0\}$. The last case above is the beginning of a cascade without investment.

Next we want to formally define **herd** as follows. Let the strategy for the investor at time t be a function $z_t(s^t)$. The equilibrium strategies are $z_t(s^t) = 1$ if $k_t(s^t) \geq 0$ and $z_t(s^t) = 0$ otherwise. Suppose there is a history of signals s^N such that $\{x_N(h_N, s_N), ..., x_0(h_0, s_0)\} \neq \{z_N(s^N), ..., z_0(s^0)\}$, where $h_t = \{x_{t-1}, x_{t-2}, ..., x_0\}, \forall t \geq 1$ and $h_0 = \emptyset$, we would say the outcome path $\{x_N, x_{N-1}, ..., x_0\}$ is a herd in that the actions taken in the private information game, given the history of signals, are not the same as the actions taken in the public information game. Furthermore, we say that the outcome path $\{x_N, x_{N-1}, ..., x_0\}$ is a herd of investment if it is a herd and there is some $t < N$ such that $x_s = 1$ for all $s \geq t$. Similarly, $\{x_N, x_{N-1}, ..., x_0\}$ is called a herd of no investment if it is a herd and there is some $t < N$ such that $x_s = 0$ for all $s \geq t$. The authors also establish the proposition that the equilibrium has both herds of investment and herds of no investment.

05/04/1999

Subject: Risky Assets Pricing Model
Reference: Lewis, Karen (1995) "Puzzles in International Financial Markets," *Handbook of International Economics*, Vol. 3, Chapter 37

Before we discuss the model, it is useful to recall a few key relationships involving interest parity. Let the home country be the US, and the rest of world Japan. Define the nominal spot exchange rate as $S = \$/¥$, and the forward rate as $F = \$/¥$. Let's further define $f \equiv (F - S)/S$. If $f > 0$, it stands for that $ is selling at forward discount, and ¥ is selling at forward premium. (The intuition behind is the following: $f > 0 \Rightarrow F > S \Rightarrow (1/F) < (1/S) \Rightarrow \text{future}(¥/\$) < \text{current}(¥/\$) \Rightarrow$ $ is selling at discount.)

Recall the covered interest parity as in notes pp. 45, we have $i - i^* = f$. Define the real exchange rate as $q = SP^*/P$, which implies $\hat{q} = \hat{s} + \pi^* - \pi$. The hats here stand for growth rate of levels, and π^*, π stand for Japan inflation rate and the US inflation rate, respectively. In expectations, the relationship above can be written as $\hat{s}^e = \pi^e - \pi^{*e} + \hat{q}^e$, designating the determinants of expected depreciation of $. For simplicity, we assume $\hat{q}^e = 0$.

Let $\bar{r}, \bar{r}^*$ be the expected return to domestic and foreign assets for sure. Then Fischer Parity tells us that $\bar{r} = i - \pi^e$ and $\bar{r}^* = i^* - \pi^{*e}$, and thus we have the real differential in yields as

[1] $\bar{r}^* - \bar{r} = i^* - i + \pi^e - \pi^{*e} = \hat{s}^e - f$.

Suppose the initial wealth level of the representative agent is W_0, and the wealth level in the next period is a random variable with expectation $\overline{W}$ and variance σ_W^2. There are two assets, one domestic and one foreign, from which the agent can choose to invest. The returns to these assets are also random variables with expectation $\bar{r}, \bar{r}^*$ and variances $\sigma_r^2, \sigma_{r^*}^2$. We assume that there is no correlation between the returns and the future wealth level, but the covariance between these two assets is σ_{rr^*}. Denote x the share of foreign asset in the agent's investment portfolio.

The representative agent's objective is to maximize the expected utility that depends upon the expectation and variance of future wealth level. That is,

$$x = \arg\max_x \{EU(\overline{W}, \sigma_W^2)\}.$$

The expected wealth level in the next period is $W = W_0[1 + xr^* + (1-x)r]$. This implies the following two relationships:

[2] $\overline{W} = W_0[(1+\bar{r}) + x(\bar{r}^* - \bar{r})]$

[3] $\sigma_W^2 = W_0^2[x^2\sigma_{r^*}^2 + (1-x)^2\sigma_r^2 + 2x(1-x)\sigma_{rr^*}]$.

The first-order-condition is

$$U_1W_0(\bar{r}^* - \bar{r}) + U_2W_0^2[2x\sigma_{r^*}^2 - 2(1-x)\sigma_r^2 + 2(1-x)\sigma_{rr^*} - 2x\sigma_{rr^*}] = 0$$

Define $\theta \equiv -2U_2W_0/U_1$, twice of the constant relative risk aversion coefficient. Then we have

$$(\bar{r}^* - \bar{r}) - \theta[x\sigma^2_{r^*} - (1-x)\sigma^2_r + (1-2x)\sigma_{rr^*}] = 0$$
$$(\bar{r}^* - \bar{r}) - \theta[x(\sigma^2_{r^*} + \sigma^2_r - 2\sigma_{rr^*}) - \sigma^2_r + \sigma_{rr^*}] = 0$$

Define $\sigma^2 \equiv \sigma^2_{r^*} + \sigma^2_r - 2\sigma_{rr^*}$, we have

$$[4]\, x = \frac{\bar{r}^* - \bar{r}}{\theta\sigma^2} + \frac{\sigma^2_r - \sigma_{rr^*}}{\sigma^2} \equiv \frac{\bar{r}^* - \bar{r}}{\theta\sigma^2} + \alpha .$$

Basically, $\sigma^2 = Var(r - r^*)$, and we call it as the relative yield variability stemming from switching one unit of assets from the foreign asset to domestic one (or the converse). We call the first component of the optimal portfolio share x as the speculative portfolio in that a higher expected differential in returns results in a higher share of investment in foreign assets, and that a higher relative yield variability results in a lower share of investment in foreign assets. The second component is called minimum variance portfolio in the sense that

$$x = \alpha = \arg\min_x \{\sigma^2_W\}.$$

Suppose in the home country there are finite number of speculators behaving according to the optimal portfolio plan as above; that is, for the j^{th} speculator, we have

$$[5]\, x_j = \left[\left(\frac{\bar{r}^* - \bar{r}}{\theta_j\sigma^2}\right) + \alpha\right].$$

Let V, V^* stand for the total supply of domestic and foreign assets. Let the total wealth level be

$$[6]\, W^T = \sum_j W_j .$$

The assets market equilibrium implies

$$[7]\ W^T = V + V^*.$$

We further denote $1/\tilde{\theta}$ the wealth weighted risk aversion coefficient,

$$[8]\, \frac{1}{\tilde{\theta}} \equiv \sum_j \frac{W_j}{\theta_j W^T}.$$

Therefore, we have

$$\sum_j \left[\left(\frac{\bar{r}^* - \bar{r}}{\sigma h^2}\right)\left(\frac{W_j}{\theta_j}\right)\right] = \frac{\bar{r}^* - \bar{r}}{\sigma^2} W^T \left[\sum_j \left(\frac{1}{\theta_j}\frac{W_j}{W^T}\right)\right] = \frac{\bar{r}^* - \bar{r}}{\tilde{\theta}\sigma^2} W^T$$

$$\sum_j x_j W_j = \left[\frac{\bar{r}^* - \bar{r}}{\tilde{\theta}\sigma^2} + \alpha\right] W^T = V^*$$

$$[9]\ \bar{r}^* - \bar{r} = \tilde{\theta}\sigma^2[V^*/W^T - \alpha].$$

That is, the expected yield differential depends upon the market risk aversion $\tilde{\theta}$, the relative yield variability σ^2 and the relative asset supply versus minimum variance portfolio α. The existence of risk premium $\bar{r}^* > \bar{r}$ depends upon $V^*/W^T > \alpha$. People will hold foreign assets more than the minimum variance share α only if there is a speculative return.

Combination of [1] and [9] implies

$$[10]\, f = \hat{s}^e - \tilde{\theta}\sigma^2[V^*/W^T - \alpha].$$

This is, the deviation of forward rate from the expected depreciation depends on the risk premium.

Furthermore, we have the following

[11] $(\hat{s}-f)=(\hat{s}-\hat{s}^{e})+(\hat{s}^{e}-f)=(\hat{s}-\hat{s}^{e})+\tilde{\theta}\sigma^{2}[V^{*}/W^{T}-\alpha]$.

This is, the excess depreciation is the combination of unanticipated news and the risk premium.

Assignment #1

Jamaica is a small open economy that produces tradables and nontradables, using sector specific capital, and labor that is mobile between sectors, but not internationally. The economy is initially in equilibrium, with external and internal balance, such that $P_T / P_N = \rho_0$ and $W / P_T = \omega_0$. However, domestic unions have just announced their intention to raise wages (in terms of tradables) to a new minimum level: $W / P_T = \omega_1 > \omega_0$. The Jamaican government is concerned about the likely effects of this change. In particular, they want to understand what will happen to Jamaica's real exchange rate, domestic income, expenditure and the current account. (They assume that it will be possible to maintain full employment.) If the minimum wage results in a current account imbalance, they want to know what policy makers could do to restore current account balance.

Your have been hired as a consultant to help them understand the implications of this minimum wage. Please write a paper (no more than 4 pages) providing your analysis. Your paper should use the following general outline:

- Introduction / Motivation (clearly set out the issues to be explored)
- Model
- Results
- Discussion

Responses to the Higher Minimum Wage

By Qin Lei

1. Introduction

The Jamaica National Workers Union just announced that they are going to raise the minimum wage to a higher level. Before this wage increase, the Jamaica government has successfully maintained both external and internal balance. It is highly relevant to the Jamaica government to find out the possible implications to some key indices of the small open economy, such as real exchange rate, domestic income, expenditure and the current account, etc., if they are trying to maintain the full employment.

This paper uses a dependent economy model with sector specific capital and mobile labor across sectors to illustrate the responses to the rising wage. In section 2, the dependent economy model is set up. We then show the implications to the key indices, both intuitively and graphically, in section 3. Section 4 concludes the paper with the policy recommendation of enhancing the productivity in the tradable sector relative to that in the nontradable sector, such as introducing outside capital into the tradable goods production sector, in order to restore the equilibrium.

2. Setup of the Model

In the supply side of the economy, suppose Jamaica produces only two composite goods, tradable T and nontradable N, the prices of which are P_T and P_N, respectively. Using the nontradable goods as *numeraire*, we can define the real exchange rate as $r = P_T / P_N$. In each sector, the factor inputs are the exogenous capital stock $\overline{K}_T$ or $\overline{K}_N$ and endogenous labor L_T or L_N. If we express the total value of the production in terms of *numeraire* goods, we have:

[1] $Y \equiv Y_N + r \cdot Y_T = Y(r; \overline{K}_T, \overline{K}_N)$.

The positive relationship between the real exchange rate and the total output is showed in Figure 1 as the YY schedule. We also assume that full employment holds in the labor market, which implies

[2] $\overline{L} = L^d(w, r) = L_T^d(w; \overline{K}_T) + L_N^d(w, r; \overline{K}_N)$,

where $\overline{L}$ is the total amount of labor resources available, w is the wage in terms of tradable, and the superscript ^{d} stands for labor demand. The relationship between the wage in terms of tradable and the real exchange rate is graphically represented by the downward sloping LL schedule in Figure 1.

In the demand side, the total expenditure E consists of demand for nontradable D_N and tradable D_T. In terms of *numeraire* goods, this is

[3] $E = D_N + r \cdot D_T$.

The market clearing condition for tradable and nontradable goods are [4] and [5], respectively.

[4] $Y_T(r) = D_T(r, E)$

[5] $Y_N(r) = D_N(r, E)$

Assume as usual that both goods are normal and the substitution effect dominates income effect, we get a positive relationship between real exchange rate and expenditure for the nontradable goods and a negative one for the tradable. Graphically, the upward sloping TT schedule stands for the clearance of the tradable goods market, and thus CA in balance; the downward sloping NN schedule represents the clearance of the nontradable goods market.

For an instance, corresponding to a real depreciation, there exist excess demand for nontradable goods and excess supply of tradable goods and thus expenditure has to be expanded to fill in the gap. However, since the expansion of expenditure will be split between tradable and nontradable goods, the same amount of expenditure expansion as the amount of increase of total output is not enough to fill in the tradable goods market gap. Therefore, the further expansion of expenditure is required to reach a CA balance. This implies a flatter slope of the TT schedule than the YY schedule. Furthermore, we know that the YY, TT and NN schedule will intersect at the same point in equilibrium, according to Walras' law.

<u>3. Responses to the Higher Wage</u>

Suppose the economy is at the equilibrium point E prior to the wage increase, and the equilibrium levels of real exchange rate and wage rate in terms of tradable are ρ_0 and ω_0, shown in Figure 2. After the implementation of the higher minimum wage $\omega_1 > \omega_0$, the full employment condition implies that only a real appreciation to the level ρ_1 could clear the labor market. That is, labor demands in both sectors are shrinking associated with the increase of wage rate, and thus only a corresponding price increase in nontradable goods would allow the nontradable sector to absorb the extra labor forces laid off from the tradable sector.

The real appreciation then implies an excess supply of nontradable goods and excess demand for tradable goods. The value of total production in terms of *numeraire* will fall to the level of Y_1. In the nontradable goods market, the expenditure has to rise to the level of E_1 to suppress the excess supply so as to keep a clearance of nontradable goods. However, the expansion of expenditure makes the situation in the tradable goods market even worse, the excess demand becomes bigger, and thus the country has to import tradable goods amount of $E_1 - Y_1$ to fill the gap between demand and supply. Therefore, the economy will stay at the dis-equilibrium point E' and experience a CA deficit amount of $E_1 - Y_1$, corresponding to the real appreciation as an aftermath of the rising minimum wage rate.

<u>4. Policy Implication</u>

In order to restore the equilibrium of the economy, we need to introduce some outside shocks into the system so that the relative productivity between two sectors will change in favor of tradable goods. One of the specific ways is to introduce some outside capital into the production of tradable goods sector. As one of the consequences, the labor forces will partly shift back to the tradable goods sector and the LL schedule will move upward associated with the positive shock. In the same time, more production of tradable goods helps to match the gap existing in the domestic tradable goods market and thus both the YY schedule and the TT schedule will shift to the right. Since there is nothing-exogenous happening in the nontradable sector, the NN schedule will remains the same. Figure 2 also illustrates that a proper extent of outside positive shocks to the tradable sector will help the economy rebuild both internal and external balances at point E', with a even higher wage rate in terms of tradable, ω_2.

All in all, the proper reaction of the policy maker, facing a higher minimum wage rate, would be introducing some positive shocks to the sector producing tradable goods and thus restoring the *CA* balances.

APPENDIX

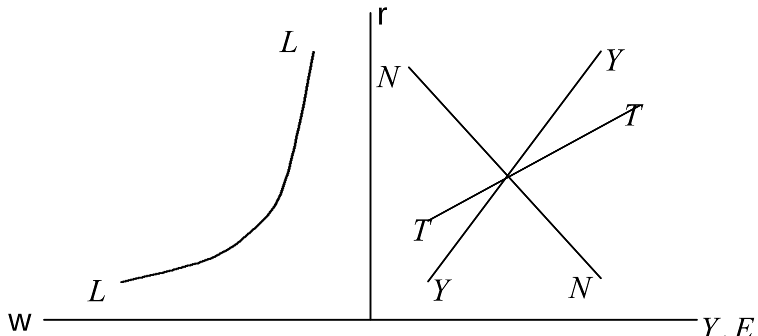

Figure 1

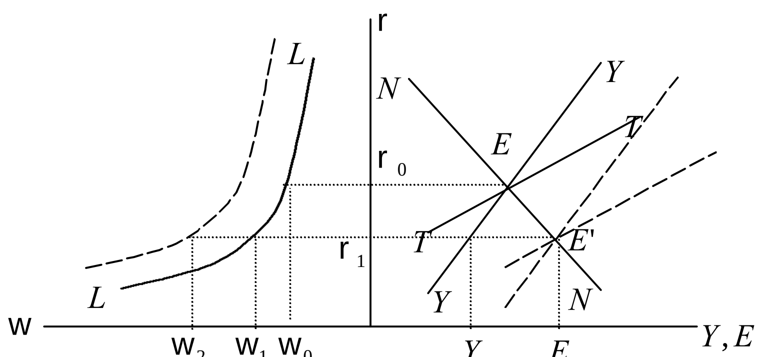

Figure 2

Assignment #2

Iceland is a small open economy. Initially, it produces and consumes composite traded good and composite nontraded goods using capital and labor. While both inward and outward migration of labor are minimal, Iceland has no restrictions on international capital movements. The country can borrow and lend at the world rate of interest, r.

Iceland has just discovered a significant reserve of titanium. This substance, which fetches a high price on world markets, can be extracted and exported, essentially using only physical capital (usage of labor inputs is minimal). While the Icelandic government is excited about the discovery, it is also concerned, in light of experiences in the U.K. and Holland following their discoveries of oil and natural gas, respectively. In particular, government officials worry that, even though titanium is unlikely to be consumed domestically, this discovery will have adverse effects on Iceland's real exchange rate and on the international competitiveness of domestic labor. The capital required for titanium production may cause capital-labor ratios to decline in the rest of the economy. They also concerned about effects on Iceland's balance of payments.

You have been hired as a consultant to help assess the likely effects of the discovery. Please write a paper (no more than 5 pages) providing your analysis. Be explicit about any assumptions you make. Your paper should use the following general outline:

- Introduction / Motivation (clearly set out the issues to be explored)
- Model
- Results
- Discussion

"DUTCH DISEASE" AGAIN?♠

BY QIN LEI

1. Introduction

It is surprising to many people in Iceland that there actually exists a huge amount of reserve of titanium in this small country, according to the latest science report. This may seem to be nothing but good news to many of us; however, the government of Iceland is worrying about the possibility of being affected by the "Dutch Disease". The so-called "Dutch Disease" comes from the experience of the U.K. and Holland following their discoveries of oil and natural gas, respectively. More specifically, the Iceland government is worrying about whether such a discovery of titanium would result in a real appreciation in this country and thus a deterioration of the international competitiveness of domestic labor. Is their worry necessary? This paper tries to shed some light on the aftermath of the new discovery; in particular, we will discuss its impact on real exchange rate, international competitiveness of domestic labor, capital-labor ratios and the balance of payment in the economy.

The paper is organized as follows. In section 2, we present a two-sector model with internationally mobile capital for the small open economy before the discovery of titanium. The model is then modified in section 3 to explain possible impacts to the economy after the new discovery. Section 4 concludes the paper.

2. Model before the Discovery

In our model, the small open economy produces two composite goods, tradable and nontradable. Using the tradable as the *numeraire* good and we define the real exchange rate as $\rho \equiv P_N / P_T$, the price of nontradable goods relative to that of tradable. Note that in this model, we allow the international mobility of capital, but not of labor; labor is allowed to freely move only across sectors. Suppose both the interest rate r in terms of *numeraire* and the price of tradable goods P_T are determined by the rest of the world. The wage in terms of tradable is w. Denote the total factor productivity in two sectors as A_T and A_N, output as Y_T and Y_N, capital inputs as K_T, K_N, and labor inputs as L_T, L_N. Two CRS production functions are given by $Y_T = A_T F(K_T, L_T)$ and $Y_N = A_N G(K_N, L_N)$, respectively. We assume further that there is a labor resource constraint due to the international immobility of labor, i.e., $\overline{L} = L_T + L_N$.

The optimization problems the firms are facing in two sectors are the following:

$$\max_{\{K_T, L_T\}} [A_T F(K_T, L_T) - wL_T - rK_T] \text{ and } \max_{\{K_N, L_N\}} [\rho A_N G(K_N, L_N) - wL_N - rK_N].$$

Note that here we can equally well write the optimization problems in the form of intertemporal optimization and then analyze the steady state of the economy. Using either setup, we should be able to derive the same set of first-order conditions as follows.

[1] $A_T f'(k_T) = r$

[2] $A_T [f(k_T) - k_T f'(k_T)] = w$

[3] $\rho A_N g'(k_N) = r$

♠ This is the revised version after evaluation.

$$[4]\ \rho A_N[g(k_N) - k_N g'(k_N)] = w$$

Here the production functions are expressed in extensive forms, and the primes stand for first order derivatives. k_T and k_N represent the capital-labor ratio in two sectors, respectively.

The capital-labor ratio k_T is thus uniquely determined in equation [1]. It is positively related to the productivity in the tradable sector and negatively related to the world interest rate. The determination of k_T then helps to determine the wage in terms of tradable in equation [2]. We also find that the wage rate is positively related to the productivity in the tradable sector and negatively related to the world interest rate. It may seem wield that the wage rate is determined solely by the tradable sector, not like by both sectors in a Dependent Economy Model; however, this is simply because we have internationally mobile capital in this model, not sector-specific one as in the Dependent Economy Model.

Now we are left with the real exchange rate ρ and k_N to be determined. From equation [3] and [4], we can graphically depict the relationship between the real exchange rate ρ and k_N in Figure 1. The intersection point of the MPK_N schedule from equation [3] and the MPL_N schedule from equation [4] would uniquely determines the real exchange rate level and the capital-labor ratio in the nontradable sector. We could further find that the impact of world interest rate on both real exchange rate and capital-labor ratio in nontradable is negative, while the impact of the productivity in the tradable sector is positive on both. The productivity in the nontradable sector has a negative impact only on the real exchange rate.

If we denote as $\overline{Q}$ the total financial wealth, which include both capital stock and foreign assets holding, in this small economy, then the total GNP could be written as $GNP = w(r)\overline{L} + r\overline{Q}$. Since in equilibrium $GNP = C_T + \rho(r)C_N$ holds, we have the following relation, denoted as GNP schedule in Figure 2,

$$[5]\ C_T = -\rho(r)C_N + [w(r)\overline{L} + r\overline{Q}].$$

In the production side of the economy, the zero-profit condition in each sector implies that in equilibrium we have

$$[6]\ Y_T = rK_T + w(r)L_T = [rk_T(r) + w(r)]L_T \equiv \xi_T(r)L_T,$$

$$[7]\ Y_N = rK_N + w(r)L_N = [rk_N(r) + w(r)]L_N \equiv \xi_N(r)L_N.$$

Using the labor resource constraint and equations [6] and [7], we have the following relation, denoted as GDP schedule in Figure 2,

$$[8]\ Y_T = \xi_T(r)\overline{L} - [\xi_T(r)/\xi_N(r)]\rho(r)Y_N.$$

We further assume the representative consumer's preference is homothetic so that we have the income expansion line in Figure 2. The economy is then locating at point E in equilibrium, and the trade surplus $Y_T - C_T$ is used to pay off the debt accumulated due to import of capital.

3. Model after the Discovery

Suppose the world price of titanium is P_τ, and define the price of titanium relative to the traditional tradable goods as $\gamma \equiv P_\tau / P_T$. Assume the production of titanium uses primarily capital and the production function is $Y_\tau = A_\tau H(K_\tau)$, where K_τ denote the amount of capital used for titanium production, and A_t is the productivity in this sector. Assume as usual that $H'(K_t) < 0$ and $H''(K_t) > 0$. Let's also assume that there is enough titanium to be extracted so that we are not considering the production capacity here.

It is apparent that the production of nontradable goods will not be affected at all in that the optimization problem remains to be

$$\max_{\{K_N, L_N\}} [\mathrm{r}\, A_N G(K_N, L_N) - wL_N - rK_N].$$

The optimization problem to the comprehensive tradable goods sector will change into

$$\max_{\{K_T, L_T, K_t\}} [A_T F(K_T, L_T) + \mathrm{g} \cdot A_t H(K_t) - wL_T - r(K_T + K_t)].$$

Note that this comprehensive optimization problem could be separated into two optimization problems: one for the traditional tradable sector,

$$\max_{\{K_T, L_T\}} [A_T F(K_T, L_T) - wL_T - rK_T],$$

and the other for the titanium production sector,

$$\max_{\{K_t\}} [\mathrm{g} \cdot A_t H(K_t) - rK_t].$$

The first-order conditions to the traditional tradable and nontradable sectors are exactly the same as before the titanium discovery, and we now have one more first-order condition for the titanium sector as $\mathrm{g} \cdot A_t H'(K_t) = r$, which will determine the optimal amount of capital used in this sector. Therefore, the real exchange rate and capital-labor ratios in two sectors will be the same as the original level. The tradable goods product wage remains the same, whereas the international competitiveness of domestic labor, measured as the real wage rate in terms of comprehensive goods, would be enhanced if titanium were actually produced.

Suppose there is no change in the total financial wealth in this economy so that the *GNP* schedule would remain the same as before. The production of titanium would shift the *GDP* schedule vertically up by the amount of the value of production of titanium in terms of traditional tradable, $\mathrm{g}Y_t = \mathrm{g} \cdot A_t H(K_t)$. In equilibrium, the economy is locating at the original point E in that the revenue from titanium production would be used to pay of the accumulated capital debt due to production of titanium. Therefore, the current account remains in balance as before.

4. Conclusion

In this small open economy, the production of newly discovered titanium could be separated from the traditional tradable sector because of the international mobility of capital. Therefore, this new discovery of titanium reserve imposes no impact on real exchange rate and capital-labor ratios in both sectors. The balance of payment would remain in balance as before and the international competitiveness of domestic labor would be enhanced. In a word, the Iceland government shouldn't worry about getting infected with the "Dutch Disease".

APPENDIX

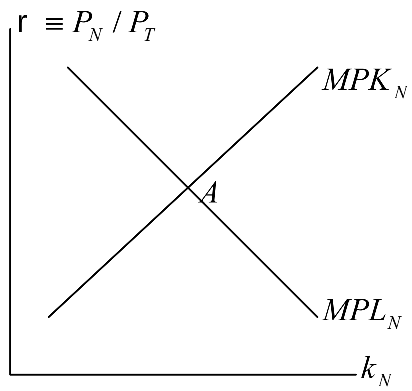

Figure 1

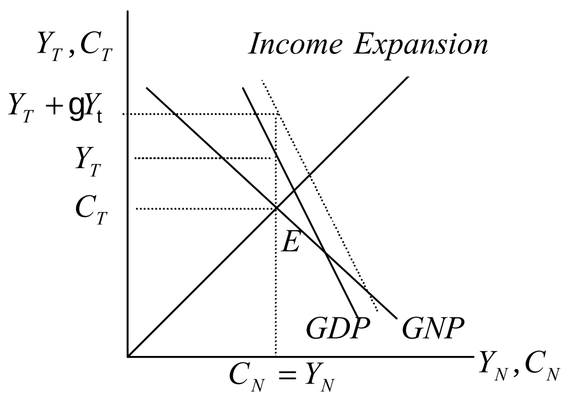

Figure 2

Practice Mid-Term Exam

I. Georgia is a small open economy that produces tradables and nontradables using capital and labor. Firms in both sectors are perfectly competitive. There are constant returns to scale in production. Tradables production is relatively capital intensive. Both factors are mobile between sectors. Capital is also internationally mobile. The world interest rate is r^*; $P_T = 1$ is the world price of tradables (and *numeraire*). Preferences are homothetic. Assume that Georgia is initially in steady state equilibrium with domestic wealth Q_0, and current account balance.

Suppose that a government is introduced. It consumes only nontradables and runs a balanced budget each period $(G = \tau)$. Assume no change in steady state domestic wealth.

a. Explain how the new government would affect wages and the real exchange rate.
b. Explain what would happen to domestic production and consumption of both goods.
c. Explain what would happen to the merchandise trade balance, and the service account balance.

[**Brief Answer**] We could use the steady-state model with mobile capital to interpret the economy. On the micro side of the economy, since the introduction of a new government would affect the household's optimal decision, the four first-order conditions remain the same. Therefore, the wages and real exchange rate would be the same as before. On the macro side of the economy, the only change occurred is that part of the private consumption of nontradable goods was shifted to the government spending. That is, both the income expansion line and the GDP line would remain the same as before, and the GNP line would horizontally shift to the left. The intersection point A between the income expansion line and the GNP line determines the amount of private consumption of nontradable goods and the amount of consumption of tradable goods. After adding the amount of government spending, we reach a point B on the GNP line that has the same vertical distance with point A. The horizontal magnitude of point B stands for the total amount of consumption and production of nontradable goods. A point C on the GDP curve that has the same horizontal distance with point B would then determine the production level of tradable goods. Since the total financial wealth hasn't been changed, the current account remains in balance while the trade balance would shrink. Suppose originally the economy was in trade surplus with service account deficit, then now there is a smaller trade surplus (or even trade deficit) with a lower service account deficit (or even service account surplus).

II. Suppose that the world is composed of two regions, the West and the East (denoted by *). There is a single tradable good, and the path of each country's output is given $(Y_0, Y_1, ...)$. There are no governments and no investment. Households can borrow and lend freely at the (endogenous) world interest rate r_t. Assume they have no debt in $t = 0$. They are characterized by Blanchard type overlapping generations, with probability of survival $\gamma = \gamma^*$. Utility functions are logarithmic. The subjective discount factors δ and δ^* are constant but may differ. Aggregate consumption in the North in time t can be shown to equal (with parallel results for the East) $C_t = (1 - \gamma\delta)W_t$, where W_t is aggregate wealth for those currently alive at home.

a. Briefly explain how to derive the equilibrium -- including current world interest rates and welfare of those who live in each region. Show the equilibrium graphically.

Suppose that a (temporary) economic crisis in the East significantly reduces Y_0^*. An analyst makes the following statement: "The reduction in Eastern income will lower world interest rate, and have a negative transmission to the West."

b. Assuming that $g = g^* = 0$, do you agree with the statement? Does your response depends on d and d^*? Explain.
c. Assuming that $g = g^* = 1$, do you agree with the statement? Does your response depends on d and d^*? Explain.

[**Brief Answer**] We ought to use the Frankel and Razin Fiscal Policy Model to explain this world. When the survival rates are zero, interest rate would not matter anymore. The $W_0^*W_0^*$ schedule would be vertical, so is the PP schedule. There will be no FF schedule in this world. The reduction of Y_0^* would horizontally shift the $W_0^*W_0^*$ to the right, but the world interest rate won't be affected and there will be no negative transmission to the West. When the survival rates are one, however, interest rate would matter. Both the $W_0^*W_0^*$ schedule and the FF schedule would shift to the right, and the PP schedule would shift to the left. Graphically there will be a lower interest discount factor, i.e., a higher world interest rate. From the equation determining the domestic wealth level, we find that a lower interest discount factor implies a lower domestic wealth level. Therefore, there is indeed a negative transmission to the West through the higher world interest rate.

III. Do you agree with the following statement? Explain why or why not. Remember that it is only your explanation that "counts".
Consider a small open economy producing tradable and nontradable goods. An expected future productivity increase will have no effect on the current account today if it is the tradable goods sector, but will cause a current account surplus today if it is in the nontradable goods sector. In either case, absorption today will rise.

[**Brief Answer**] We ought to use the Dornbusch model to tackle the problem. An expected future productivity increase in the tradable goods sector won't shift the NN schedule or CC schedule in that the production of tradable goods in this small economy won't affect the world determined tradable goods price. Therefore, the economy would stay at the original place. The higher permanent income would imply a higher current consumption both at present and in the future. It also implies a current account deficit today financed by a series of current account surplus in the future. An expected permanent future productivity increase in the nontradable goods sector would shift the NN schedule to the left. As we have discussed in class, the case of perfect substituter would result in a current account surplus at present, while the case of perfect smoother would result in a current account deficit at present. The absorption today won't necessarily rise.

Mid-term Exam

I. Consider a small open economy that produces tradable and nontradable goods using labor and capital. Both sectors are perfectly competitive. Labor is mobile between sectors and wages adjust to maintain full employment. The price of tradable goods is determined in the world market.
A. In the short run, there is no international mobility of either capital or labor, and the amount of capital in each sector is fixed. The economy initially is in equilibrium with current account balance. Explain what would happen to this (short run) equilibrium if a change in domestic tastes increased the demand for tradable relative to nontradable goods. You may assume that current account balance is maintained.
i. What are the implications for the real exchange rate and domestic wages (in terms of traded goods)?
ii. What would happen to domestic income and expenditure, the allocation of labor between sectors and domestic production of each good?
B. In the long run, labor is not internationally mobile but capital is. The country can borrow and lend at the world rate of interest, r^*. Domestic households are characterized by homothetic preferences. The country is initially in steady state equilibrium, where total consumption each period equals the value of national income. However, it has a deficit in its service account.
Explain what would happen to this (steady state) equilibrium if a change in domestic tastes increased the demand for tradable relative nontradable goods? You may assume that there is no change in the country's steady state wealth level.
i. What are the implications for the real exchange rate, domestic wages and the capital labor ratio in each sector?
ii. What are the implications for domestic production and consumption of tradable and nontradable goods?
iii. What would happen to the components of the steady state balance of payments? (The current account, the capital account, the trade balance and the service balance)
C. Are the short-run (part A) and the long-run (part B) implications for the real exchange rate the same or different? Explain briefly.

[**Brief Answer**] We ought to use the Dependent Economy Model to interpret the economy in the short run. The YY schedule won't shift corresponding to the taste switch. The TT schedule would shift to the left and the NN schedule would shift to the right. Comparing the new equilibrium to the old one, there is a real depreciation in this economy and a lower wage in terms of tradable. Both the domestic income and expenditure would expand. To explain the economy in the long run, we could use the steady-state model with mobile capital. On the micro side, it is apparent that the firm's optimization problem won't be affected by the taste switch, and thus the real exchange rate, domestic wages and capital-labor ratios in two sectors won't be affected. On the macro side, only the income expansion line would rotate to the left. Corresponding to this new steady-state equilibrium, there will be a higher production level of tradable and lower production of nontradable. The trade surplus would be enlarged in proportion to the increase of service deficit, while the current account still remains in balance, given the assumption that the steady state wealth level remain unchanged.

II. Consider a small open economy producing tradable and nontradable goods. It can borrow freely at a world interest rate r^*. Initially $r^* = \delta$ (the subjective discount rate). The country has no initial international debt or assets and is in steady state with current account equal to zero. Suppose there is a 10% decline in r^* at the current period only.

Do you agree with each of the following statements? In each case, explain why or why not. Remember that it is your explanation that "counts".
A. The 10% temporary fall in world interest rate is likely to cause a more than 10% decline in the interest rate relevant for domestic consumption and saving decisions.
B. As a result of the temporary change in world interest rates, the current account this period could go into surplus, balance or deficit -- depending on household preferences.

[**Brief Answer**] We use the Dornbusch model to explain this problem. The CC schedule would shift to the right so that the current consumption level is higher relative to the future, and the future price would fall relative to current level. The higher demand for nontradable goods would imply a higher nontradable price relative to the tradable goods, therefore there would be a lower future price relative to current level, or a higher effective real interest rate. That is, the presence of a nontradable sector would dampen the impact of changes in international interest rates through the relevant interest rate. With zero initial debt, the current income won't change at all. Under the case of certain substitution, the higher current consumption would imply a current account deficit at present. For the case of perfect substituter or smoother, the economy locates at the same equilibrium point and zero initial debt implies current account in balance.

III. Suppose that the world is composed of two regions, the North and the South (denoted by *). There is a single tradable good, and the path of each country's output is given $(Y_0, Y_1, ...)$. There is no investment. Governments in both regions initially set $G = T$ (they spend exactly the amount collected in taxes). Households can borrow and lend freely at the (endogenous) world interest rate r_t. Assume they have no debt in $t = 0$. They are characterized by Blanchard type overlapping generations, with probability of survival $g = g^* > 0$. Utility functions are logarithmic. The subjective discount factors d and d^* are constant but may differ. Aggregate consumption in the North in time t can be shown to equal (with parallel results for the North) $C_t = (1 - gd)W_t$, where W_t is aggregate wealth for those currently alive at home.
Suppose the government in the North decides to spend more today, without changing the amount of taxes collected in any period.
A. Explain what would happen to world interest rates today.
B. Explain what would happen to the wealth of those currently alive in the North.
C. Is the policy change in the North transmitted abroad? Explain briefly.

[**Brief Answer**] We use the Frankel and Razin Fiscal Policy Model to tackle the problem. The higher government spending today must be financed by a lower future government spending. The $W_0^* W_0^*$ schedule remains the same, while the FF schedule shifts to the right and the PP schedule shifts to the left. The world interest factor would be lower, i.e., a higher world interest rate. From the equation determining the domestic wealth level, we know a lower interest factor would imply a lower domestic wealth level. It is obvious that the policy change in the North has a negative impact on the South transmitted through a higher world interest rate.

Assignment #3

Hungary is a small open economy producing a single composite traded good. It has extensively trade and financial ties to the European Union (EU). The Hungarian koruny floats freely against the euro. (S = koruny/euro) The euro was established in January 1999 when major members of the EU initiated a monetary union (the EMU). The euro is managed by the European Central Bank (ECB). Other EU member currencies are allowed to circulate until January 1, 2002, after which only the euro will be legal tender. Suppose that, on January 1, 2000 the governor of the ECB announces that he believes the ECB should permanently tighten monetary policy beginning on January 1, 2002.

Hungary policy makers have asked you to assess and explain the likely implications of this announcement for their economy. Explicitly, what is likely to happen to Hungarian prices, nominal interest rates and the real and nominal value of the koruny? Please write a paper (no more than 5 pages!!) providing your analysis. Your paper should use the following general outline, and may include graphs:

-- Introduction/Motivation (clearly set out the issues to be explored)
-- Model
-- Results
-- Discussion

You may assume that for Hungary, output is exogenous, and purchasing power parity, and uncovered interest parity hold. For the EU, Fischer interest parity holds. Money demands in the two countries can be described as:

Hungary: $m_t - p_t = -\eta i_{t+1} + \phi y_t$

EU: $m_t^* - p_t^* = -\eta i_{t+1}^*$

THE ECONOMIC CLOUT OF EU: ONE SIMPLE APPLICATION♠

By Qin Lei

1. Introduction

As a small open economy producing a single composite traded goods, Hungary extensively trades with the newly established European Union (EU). Currently both the currencies of the EU member countries and the EU official currency euro are circulating, and it is well known that by January 1st 2002 only euro will be the legal tender within EU. Although it is the Hungarian government's policy to maintain its Hungarian currency, koruny, floating freely again the euro, the government is interested in knowing how this newly established EU would affect its own economy. Specifically, suppose that on January 1st 2000 the governor of the European Central Bank (ECB) announces that he believes the ECB should permanently tighten monetary policy beginning on January 1st 2002. What impact on the domestic economy can the Hungarian government expect from such a monetary policy change in EU? This paper tries to use a simple monetary model to interpret possible consequences on Hungarian prices, nominal interest rates and the real and nominal exchange rate of the koruny.

2. What will happen in the EU?

Before we explain the impact on the Hungarian economy from the change in monetary policy in the EU, it is beneficial to understand what will happen within the EU. We use a very simple system here to delineate the economy within the EU; that is, the Fischer interest parity holds and the real money demand elasticity with respect to nominal interest rate is constant at η. In logarithms, the economy could be expressed in the following two equations.

[1] $i^*_{t+1} = r^* + E_t p^*_{t+1} - p^*_t$

[2] $m^*_t - p^*_t = -\eta i^*_{t+1}$

Here i^*_{t+1}, r^*, p^*_t, and m^*_t are nominal interest rate, constant real interest rate, price and money supply. $E_t p^*_{t+1}$ is the expected next period price given information available at the current period.
Combining these two equations, we can find the following forward solution of prices within the EU, provided that the condition of non-speculative bubbles holds,

$$p^*_t = \frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i E_t(m^*_{t+i} + \eta r^*).$$

Denote the current time period as 0, the time period of announcement as 1 and the time period of enforcement of the new tightened money supply as 3. For simplicity, let $r^* = 0$, $m^*_t = \overline{m}_A \text{ (for } t < 3)$ and $m^*_t = \overline{m}_B \text{ (for } t \geq 3)$, where $\overline{m}_A > \overline{m}_B$. Therefore, we have the impact on price level within EU from the change in money supply as follows:

$$p^*_t = \begin{cases} \overline{m}_A & (t=0) \\ \overline{m}_A + [\eta/(1+\eta)]^{3-t}(\overline{m}_B - \overline{m}_A) & (t = 1,2). \\ \overline{m}_B & (t \geq 3) \end{cases}$$

♠ This is the revised version after evaluation.

Plugging this solution into equation [2], we have the impact on the nominal interest rate within EU from the change in money supply as follows:

$$i_{t+1}^{*} = \begin{cases} 0 & (t=0) \\ (1/\eta)[\eta/(1+\eta)]^{3-t}(\overline{m}_B - \overline{m}_A) & (t=1,2). \\ 0 & (t \geq 3) \end{cases}$$

3. What will happen in Hungary?

To better explain the issues related to exchange rate, interest rate and price level in a small economy such as Hungary, we assume that within this economy both purchasing power parity and uncovered interest parity hold. These two parities, in logarithms, could be written as

[3] $p_t = s_t + p_t^{*}$, and

[4] $i_{t+1} = i_{t+1}^{*} + E_t s_{t+1} - s_t$.

We further assume that output y_t in this economy is exogenously given, and the money market equilibrium condition, in logarithms, is

[5] $m_t - p_t = -\eta i_{t+1} + \phi y_t$.

Note that all variables without asterisk are ones in Hungary corresponding to those in EU. s_t is the logarithm of nominal exchange rate defined as the number of units of koruny per euro.

Combination of [3], [4] and [5] would imply the following forward-looking solution of the nominal exchange rate level, assuming the condition of non-speculative bubbles holds,

$$s_t = \frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i E_t[m_{t+i} + \eta i_{t+i+1}^{*} - p_{t+i}^{*} - \phi y_{t+i}].$$

From equation [2], we know $\eta i_{t+i+1}^{*} - p_{t+i}^{*} = -m_{t+i}^{*}$. This helps to reduce the solution above into

$$s_t = \frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i E_t[m_{t+i} - m_{t+i}^{*} - \phi y_{t+i}].$$

For simplicity, let's assume $y_t = \overline{y}_H = 0$ and $m_t = \overline{m}_H$ for all the time, then we have

$$s_t = \begin{cases} \overline{m}_H - \overline{m}_A & (t=0) \\ \overline{m}_H - \overline{m}_A - [\eta/(1+\eta)]^2(\overline{m}_B - \overline{m}_A) & (t=1) \\ \overline{m}_H - \overline{m}_A - [\eta/(1+\eta)](\overline{m}_B - \overline{m}_A) & (t=2) \\ \overline{m}_H - \overline{m}_B & (t \geq 3) \end{cases}.$$

Plugging the solutions to s_t and p_t^{*} into equation [3], we have $p_t = \overline{m}_H$. Plugging this into equation [5], we have $i_{t+1} = 0$.

4. Interpretation of the results

The change in monetary policy in EU and its consequences on its own price and nominal interest rate are presented in Figure 1 and Figure 2. It is clear that before the announcement both money supply m_t^{*} and price level p_t^{*} were stable at $\overline{m}_A$, and the nominal interest rate within EU is zero. Immediately at the period of announcement, and henceforth until the enforcement of the new monetary policy, the price level within EU will fall due to the expectation of a permanent lower money supply in the future. Thus the nominal interest rate has to fall accordingly in order to restore the money market equilibrium. After the enforcement of the new monetary policy, both money supply and price level will be stable at a lower level $\overline{m}_B$, and the nominal interest rate will go back to zero.

The aftermath to the Hungarian economy is presented in Figure 3. By assumption, the domestic money supply is constant at $\overline{m}_H$ throughout the time. Because of the same money demand elasticity of interest rate within both EU and Hungry, the domestic price level stays stable at the same level of the constant money supply level. This also implies the constancy of domestic interest rate. The purchasing power parity and uncovered interest parity force the nominal exchange rate changes against the interest rate within the EU. As we analyzed above, the interest rate in EU will drop at first and then go back to zero level as before once the monetary policy is in place. That is, there is a sustained nominal depreciation until the enforcement of new policy and henceforth the nominal exchange rate remains constant.

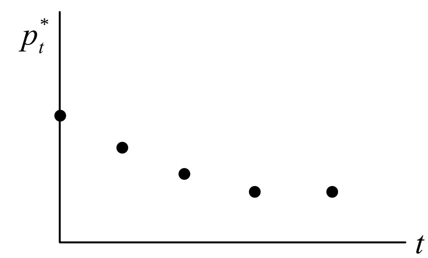

Figure 1

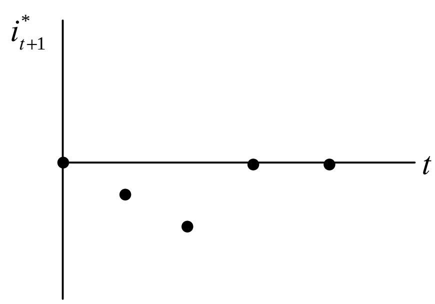

Figure 2

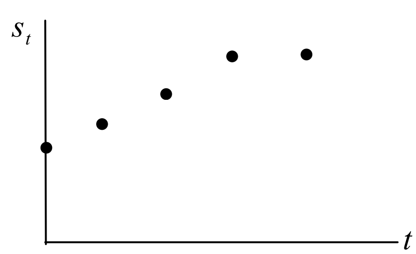

Figure 3

Practice Final Exam

I. Bulgaria is a small open economy that fixes its currency (the leva) to the dollar. Assume that this exchange regime is credible. The following equations describe the economy:

(1) $m_t - p_t = -\eta i_t + \phi \bar{y}$ Money market equilibrium

(2) $p_t = s_t + p^*$ Purchasing power parity

(3) $i_t = i^* + (E_t s_{t+1} - s_t)$ Uncovered interest parity

Variables here are in logs, defined as usual from class. Assume $p^* = i^* = 0$ and output is exogenous and fixed.

A. What monetary policy is consistent with maintaining $s = \bar{s}$? Provide an explicit solution, and discuss it briefly.

B. There is an unanticipated, permanent decline in Bulgarian output. What are the implications for current monetary policy, prices and interest rates?

C. Would you answer to part **B** have been different if Bulgarian output were expected to decline permanently, beginning in two years?

D. Suppose the market now believes there is a positive probability of a devaluation, as shown in equation (4) below. Discuss implications for monetary policy, prices and interest rates, assuming the exchange rate remains fixed.

(4) $E_t s_{t+1} = \gamma \bar{s}$, where $\gamma > 1$.

[**Answer**]

A. The credible fixed exchange rate reduces equation (3) into $i_t = 0$. Equation (2) implies $p_t = \bar{s}$, and thus equation (1) implies $m_t = \bar{s} + \phi \bar{y}$. That is, the money supply has to be fixed at a level determined by the fixed exchange rate, the exogenous output level and the money demand elasticity to output.

B. From the solution in part **A**, we know an unexpected decline in output forces the money supply fall to a lower level, but there is nothing changed in prices and interest rates.

C. Because of the fixed exchange rate regime, there is no role of future in determining the interest rate. Therefore, the impact on the economy will be the same as in part **B**.

D. Combination of equations (3) and (4) implies $i_t = (\gamma - 1)\bar{s} > 0$, and thus $m_t = [1 - \eta(\gamma - 1)]\bar{s} + \phi \bar{y}$. The money supply has to be lower than the level in part **A**, due to the positive probability of devaluation. The interest rate will be higher and prices remain the same as the fixed exchange rate.

II. Do you agree with each of the following statements? In each case, explain why or why not. Remember it is your explanation that counts.

A. If the German deutsche mark (DM) is expected to appreciated by 5% in nominal terms relative to the US dollar ($) between 1998 and 1999, then there will be a 5% forward premium on the DM relative to the dollar.

B. Consider a small open economy with high capital mobility. Prices are sticky in the short run. A tax cut will tend to cause a current account deficit if the exchange rate is flexible, but a current account surplus if the exchange rate is fixed.

[**Answer**]

A. First of all, let's define the nominal exchange rate as $S \equiv DM/\$$, and German is considered as the home country. The forward premium is defined as $f \equiv (F - S)/S$. If $f < 0$, then DM is more expensive in the future than today, i.e., DM is selling at a forward premium. The original statement is saying when $\hat{s}^e = -5\%$, we have $f = -5\%$. In order to verify this relationship, we use a few conclusions from the risky assets pricing model.

$$\bar{r}^* - \bar{r} = (i^* - i) + (\pi^e - \pi^{e*}) = \hat{s}^e - f = \theta\alpha[(V^*/W^T) - \alpha]$$

This tells us that unless the share of risky assets in the total wealth reaches the minimum variance share, $\hat{s}^e \neq f$. The original statement is not necessarily true.

B. It is apparent that we can use IS-LM-BP model to explain this statement. The system of equations is listed as follows.

$$IS: Y = C(Y) + I(i) + G - I + NX(SP^*/P, Y)$$

$$LM: \frac{M}{P} = \frac{DC + R}{P} = L(i, Y)$$

$$BP:\ NX(SP^*/P, Y) + CAP(i - i^*) = \Delta R = 0$$

Suppose the economy originally rests at point A in Figure 1 and 2. The tax cut implies higher output demand so that the IS curve should shift to the right, and the economy reaches point B with a higher interest rate and output. Because of the high capital mobility, the impact of interest rate effect on the BOP dominates and thus there is a BOP surplus at point B.

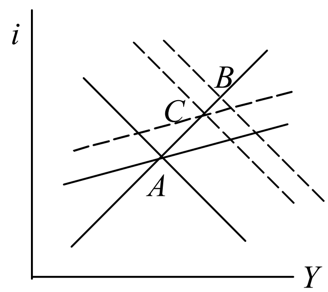

Figure 1

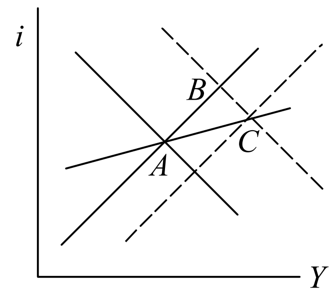

Figure 2

For the case of flexible regime, in Figure 1, there must be a nominal appreciation in order to bring the BOP back into balance. However, the nominal appreciation tends to shift both IS and BP curve to the left, and the shift of BP curve is larger. The economy will eventually locates at point C, standing for a fiscal policy less effective than in a closed economy. Both the nominal appreciation and the higher output level imply the current account deficit.

For the case of fixed regime, in Figure 2, the reserve level has to grow corresponding to the BOP surplus. The higher reserve level will shift the LM curve to the right until the economy reaches point C, standing for a fiscal policy more effective than in a closed economy because of the reinforcing monetary expansion. The higher output level then implies the current account deficit.

Therefore, the current account experiences deficit regardless of the exchange rate regime, and the original statement is false.

III. Consider a small open economy with a flexible exchange rate ($s = peso/\$$, where pesos are the domestic currency) and free capital mobility. There is perfect foresight about domestic inflation and exchange rate depreciation. Domestic residents hold two assets: domestic money (M) and foreign assets. Each foreign asset yields \$1 (in real terms) per period indefinitely. The number of foreign assets is F and F/r is the real value of these claims on the rest of the world. r is the fixed real interest rate.

You are asked to discuss exchange rate dynamics for this economy using the equations below. (Note that a dot over a variable denotes a time derivative.)

(1) $P = SP^*$ (assume $P^* = 1$)	Purchasing power parity (note $\dot{P}/P = \dot{S}/S$)
(2) $M/P = g(r + \dot{S}/S)\cdot W$ ($g' < 0$)	Portfolio balance/money market equilibrium
(3) $W = M/P + F/r$	Real wealth
(4) $Z = Z(W, r)$ ($Z_W < 0,\ Z_r > 0$)	National saving
(5) $I = I(r)$ ($I_r < 0$)	National investment

A. The asset market equilibrium:
a. Derive the relationship between the exchage rate(s) and the stock of foreign assets (F) such that $\dot{s} = 0$. Graph the relationship and explain it in words.
b. Is the economy always on this schedule? If so, explain why; if not, explain what it means to be off the schedule.
B. The external balance:
a. Derive the current account balance schedule. Graph it and describe the relationship in words.
b. Is the economy always on this schedule? If so, explain why; if not, explain what it means to be off the schedule.
C. Suppose you observe this economy from an initial position of current account deficit. Describe its adjustment over time graphically and in words. What happens to the exchange rate, the current account and wealth along the adjustment path?
D. Show and explain what would happen if there were a permanent, unanticipated 10% contraction of the nominal money supply this year (in 1998).
E. Show and explain what would happen if it were announced in 1998, that the money supply would be contracted permanently by 10% in January 2000.

[**Answer**]
A. Combination of (1), (2) and (3) implies

$$\frac{M/S}{M/S + F/r} = g(r + \dot{S}/S).$$

Define

$$h(\cdot) \equiv g^{-1}\left[\frac{M/S}{M/S + F/r}\right],$$

Then we have

$$\dot{S}/S = h(\cdot) - r,$$

where $h'(\cdot) < 0$ because a higher opportunity cost induces a lower money holding.

Hence $\dot{S} = 0 \Rightarrow h(\cdot) = r$ and we can write $\dot{S} = a(S, F; M, r) = a(+,+;-,-)$. Graphically this is a downward sloping curve in Figure 3, standing for the locus of points where the opportunity cost of holding money is constant and zero inflation. The economy may not necessarily stay on this curve all the time in that the exchange rate may rise or fall.

B. Using the accounting identity that current account is the net feign assets holding, we have

$$\dot{F} = Z(W, r) - I(r) = Z(M/S + F/r, r) - I(r).$$

Hence $\dot{F} = 0 \Rightarrow Z(M/S + F/r, r) - I(r) = 0$ and we can write $\dot{F} = b(S, F; M, r) = b(+,-;-,+)$. Graphically this is an upward sloping curve in Figure 3, standing for the locus of points where current account is in balance. The economy may not necessarily stay on this curve all the time in that the current account may experience surplus or deficit.

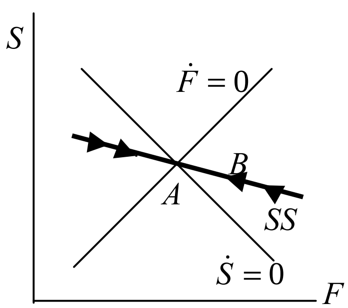

Figure 3

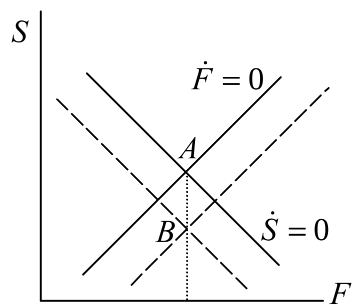

Figure 4

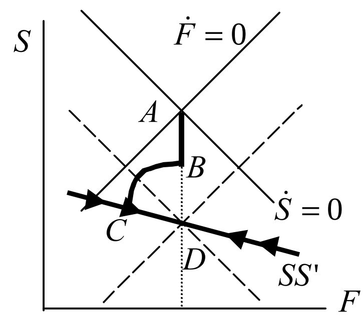

Figure 5

C. It is easy to determine that $\dot{S} = 0$ curve is exploding and $\dot{F} = 0$ curve is stable. This implies the stable arm SS as depicted in Figure 3. We need to determine a point on the stable arm that stands for a current account deficit. The current account deficit means $\dot{F} < 0$ and we know any point below the $\dot{F} = 0$ curve will satisfy such a property. Hence we choose a representative point B that is below the steady state point A. The adjustment along the stable arm from point B to point A corresponds to a nominal depreciation and shrinking current account deficit, and the wealth level is falling over time.

D. Corresponding to the unanticipated contraction of money supply, both the $\dot{S} = 0$ curve and $\dot{F} = 0$ curve would shift downward at the same amount for given F. The new steady state will be point B in Figure 4. There will be a nominal appreciation and no change in foreign assets holding.

E. Corresponding to the anticipated contraction of money supply, we know from part **D** that the eventual steady state will be point D in Figure 5. At the moment when news comes out, the foreign assets holding is sticky and only a nominal appreciation up to some point like B would possibly bring the economy to the new steady state. From then on and until the moment when the new monetary policy is effective, the economy will evolve to some point C following the forces of the original dynamic system. At the moment when the new policy is effective, the economy is at point C, which is on the stable arm for the new dynamic system. From then on, the economy will adjust along the new stable arm until the new steady state.

IV. Consider a small open economy that produces tradables (T) and nontradables (N) using capital and labor. There are constant returns to scale in production in both sectors. N is relatively labor intensive. Both sectors are competitive.

(1) $Y_T = A_T F(K_T, L_T)$

(2) $Y_N = A_N F(K_N, L_N)$

In the short run, labor is mobile between sectors, capital is sector specific and there is no international factor mobility. Real exchange rates adjust to maintain current account balance. In the long run, labor and capital are mobile between sectors, and capital is internationally mobile, with a fixed real world interest rate r. The current account is balanced.

A. How is the real exchange rate typically measured in this context?

Consider a permanent increase in productivity in the nontradable goods sector.

B. How would this affect the real exchange rate and real wages in terms of tradables in the short run?

C. How would this affect the real exchange rate and real wages in terms of tradables in the long run?
D. Based on your analysis above, are the short run and the long run implications the same or different? Discuss briefly.

[**Answer**]
A. Typically we define the real exchange rate as $r \equiv P_T / P_N$. Here we adopt this notation for the short run analysis and use $r^* = P_N / P_T$ for the long run analysis.

B. **C**. **D**. We could use the dependent economy model to analyze the economy in the short run, and use the steady-state model to analyze the economy in the long run. The answer is very similar to the one to the mid-term exam.

Final Exam

I. Do you agree with each of the following statements? In each case, explain why or why not. Remember -- it is your explanation that "counts."

A. Consider a small open economy producing both tradable and nontradable goods. A rise in the capital stock used to produce tradables would cause a real exchange rate appreciation. This rise in the relative price of nontradables would increase production of nontradables.

B. Assume that the Japanese yen is selling at a 5% discount (relative to the U.S. dollar) in the one-month forward market and that market participants expect the yen to depreciate by 10% relative to the dollar over the next month. Then the expected real return on Japanese assets is %% more than the expected real return on U.S. assets and there is an "excess supply" of Japanese assets.

C. An investment boom will cause a smaller interest rate increase and a larger output expansion in a closed economy tan in a small open economy with fixed exchange rates.

D. An increase in government spending now (to pay for the war in Kosovo) would cause a greater deterioration in the 1999 U.S. current account deficit if it were expected to be offset by future government spending cuts instead of tax changes, and if households were infinitely lived.

II. Consider a small open economy producing tradables and nontradables. Its households are infinitely lived, and can borrow or lend at a world real interest rate r^*. It has no government or investment. Initially, it has a balanced current account.

A. Let r^c be the interest rate relevant for domestic consumption and saving decisions. Explain briefly why r^c need not be equal to r^*. When will $r^c > r^*$?

B. Do you agree with the following statement? Explain why or why not:

"Suppose it is announced today that, next year, there will be a permanent productivity improvement in the nontradable sector. Then if r^c rises, there will be a current account surplus this year, and if r^c falls, there will be a current account deficit this year."

III. Barbados is a small open economy. The value of its currency, the Barbados dollar, is fixed to the U.S. dollar at $\bar{S} = 2B\$/US\$$. It is well known that if the central bank runs out of foreign exchange reserves, it will allow the currency to float. In the following equations, all variables (except the interest rate i) are given in logs.

(1) $m_t - p_t = -\alpha i_t$ real money demand

(2) $m_t = d_t + r_t$ money supply = domestic credit + reserves

(3) $p_t = p^* + s_t$ PPP holds

Domestic monetary authorities are increasing domestic credit at a constant rate:

(4) $\dot{d}_t = \mu$.

Uncovered interest parity does not hold, because there is a risk premium ($\gamma > 0$) on domestic assets: (5) $i_t = i^* + E(\dot{s}_t) + \gamma$.

Assume that $i^* = p^* = 0$.

A. Assume the fixed exchange rate is fully credible initially. Using the equations above, show algebraically and explain briefly how the following variables would evolve: m, d, r, i. Does it make sense to assume that the fixed rate will remain fully credible?

B. Solve for the shadow exchange rate, $\tilde{s}$, that would exist if $r = 0$ and the exchange rate were allowed to float freely. Does your answer depend on the risk premium? Explain.

C. Will Barbados have a balance of payments crisis? If not, explain why not. If it will: solve for the time at which the fixed exchange rate will collapse, and explain what the collapse time depends on. How, if at all, does the risk premium matter?

IV. Malta is a small open economy producing a single tradable good. Its currency, the liri, floats freely but domestic prices are sticky. The central bank controls the money supply. There is perfect foresight. The following equations describe key features of the economy: (All variables except interest rates are in logs)

(1) $i_t = i^* + ({}_t s_{t+1} - s_t)$	uncovered interest parity, where $i^* > 0$
(2) $m_t - p_t = -\alpha i_t + \phi y$	real money demand, where $0 < \phi < 1, \alpha > 0$
(3) $m_t = \overline{m}$	constant money supply
(4) $y_t^d = \overline{y} + \delta(q_t - \overline{q})$	output demand, where $0 < \delta < 1$

Here, $q_t \equiv s_t + p^* - p_t$ is the real exchange rate and $\overline{q}$ is defined so that $y_t^d = \overline{y}$ (full employment output) when $q_t = \overline{q}$.

The domestic price level adusts graduatlly according to equation (5):

(5) $p_{t+1} - p_t = \psi(y_t^d - \overline{y}) + (\tilde{p}_{t+1} - \tilde{p}_t)$, where $0 < \psi < 1$ and $\tilde{p}_t \equiv s_t + p^* - \overline{q}$.

Assume $p^* = \overline{y} = 0$, but $i^* > 0$.

A. First, solve for the change in Malta's real exchange rate, $\Delta q = q_{t+1} - q_t$, as a function of the levels of the nominal and real exchange rates. Graph the $\Delta q = 0$ schedule. Explain and show what happens when the economy is off this schedule.

B. Next, solve for the change in Malta's nominal exchange rate, $\Delta s = s_{t+1} - s_t$, as a function of the levels of the nominal and real exchange rates. (Be explicit about any additional assumptions you make.) What else does Δs depend on? Graph the $\Delta s = 0$ schedule. Explain and show what happens when the economy is off this schedule.

C. Using your responses above, solve for the steady state levels of s and q. Graph and discuss the steady state, and the system's dynamics away from its steady state equilibrium. (Assume that the economy always eventually ends up at its steady state.)

D. Suppose that Malta is initially in steady state. There is a permanent, unanticipated increase in foreign interest rates ($r^{*}{'} > i^*$). Show and explain what would happen immediately and over time to nominal and real exchange rates, domestic income and domestic interest rates. Would the exchange rate overshoot? Explain why or why not. Is this what you would expect to happen in a small open economy when foreign interest rates rise? Briefly discuss which aspects of these results you find realistic, and which you find unrealistic. Can you suggest any changes in the model that would be likely to make the results more realistic?