Momentum is Not an Anomaly

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Abstract

In this paper, we develop a new approach to test whether momentum is indeed an anomaly in that it reflects delayed reactions, or continued overreactions, to firm specific news. Our methodology does not depend on a specific model of expected returns and, more importantly, does not require a decomposition of momentum profits. Yet we provide distinct testable predictions that can discriminate between the two diametrically opposed causes for the profitability of momentum strategies: time-series continuation in the firm-specific component of returns, and cross-sectional differences in expected returns and systematic risks of individual securities. Our results show that, contrary to the common belief in the profession, momentum is not an anomaly; we find no evidence of continuation in the idiosyncratic component of individual-security returns. The evidence is instead consistent with momentum being driven entirely by cross-sectional differences in expected returns and risks of individual securities.

1 Introduction

The proposition that the idiosyncratic component of individual firms' returns displays continuation now appears to be widely accepted among academics and practitioners. Evidence of this acceptance can be found in the proliferation of papers that model individual asset prices in ways that generate positive serial covariance in idiosyncratic asset returns.[1] The literature has drawn this conclusion based on the findings of Jegadeesh and Titman (1993) and numerous subsequent papers. Jegadeesh and Titman show first that buying extreme winners and selling extreme losers generates annualized raw returns of 8% to 18% over three- to 12-month holding periods. Second, the profitability of these strategies is robust across firms of different market capitalizations and CAPM betas. Finally, the authors conduct tests to establish that the profitability of momentum strategies is driven entirely by the firm-specific component of an individual asset's return. The main conclusion of their study is that the "...evidence is consistent with delayed price reactions to firm-specific information."

The remarkable aspect of momentum is that it has proven to be both empirically robust and to defy a rational explanation. Fama (1998) and Barberis and Thaler (2003) reflect the general consensus in the literature when they pronounce momentum to be an anomaly worthy of a behavioral explanation. Similarly, Jegadeesh and Titman (2005), after reviewing the momentum literature, conclude that "...momentum effect is quite pervasive and is very unlikely to be explained by risk." This is particularly surprising because, unlike many anomalies, momentum immediately lends itself to a simple economic explanation. As has long been recognized, strategies that rely on *relative strength* should be expected to earn positive returns due to cross-sectional variation in expected returns of individual securities. Since, on average, stocks with relatively high (low) returns will be those with relatively high (low) expected returns, a momentum strategy should on average earn positive profits. Thus, cross-sectional variation in either unconditional or conditional mean returns should contribute to the profitability of momentum strategies.[2] The key element of this simple explanation is that momentum is a cross-sectional phenomenon and it attributes little, if any, of the strategy's profits to time-series predictability in the idiosyncratic component of individual stock returns.

In this paper, we revisit this explanation for the profitability of momentum trading strategies. We do so by proposing an alternative methodology for ascertaining if profits to momentum strategies derive from idiosyncratic components. Importantly, we bypass the seemingly critical step of modeling and, more importantly estimating, the mean returns of securities. This key feature of our

[1] As examples, see de Long, et al. (1990), Daniel, Hirshleifer, and Subrahmanyam (1998), Barberis, Shleifer, and Vishny (1998), Hong and Stein (1999), and Wu (2007). Several empirical papers investigate this issue; among several others, see Chan, Jegadeesh, and Lakonishok (1996), Lee and Swaminathan (2000), and Hou and Moskowitz (2005). Our collective acceptance that momentum is an anomaly is also signaled by studies that attempt to gauge whether the frequent turnover involved in relative strength strategies can withstand the transactions costs inherent to such strategies [see, for example, Korajzyck and Sadka (2004) and Sadka (2006)].

[2] Cross-sectional variation in unconditional mean returns as a source of momentum profits is investigated in Conrad and Kaul (1998). Berk, Green, and Naik (1999), Chordia and Shivakumar (2002), Cochrane (1996), Johnson (2002), Lewellen (2002), and Liu and Zhang (2007) investigate conditional variation in mean returns.

modeling and methodology makes our inferences reliable and immune to model misspecification and measurement errors that plague past studies. Specifically, we recognize that attempts to determine the profit sources of the traditional momentum strategy, which classifies individual securities as winners and losers, suffer from a classic identification problem. Namely, we can only observe the total momentum profits and, consequently, decomposing them into rational and idiosyncratic components requires the specification and estimation of a model for mean returns. We instead impose a minimal set of restrictions on the return generating process, and show that the pattern in expected momentum profits using *portfolios* of especially a small number of securities will be distinctly different if profits arise from cross-sectional variation in mean returns and risks versus time series variation in idiosyncratic returns. The key is to implement momentum strategies with winner and loser portfolios, as opposed to winner and loser individual stocks.

Specifically, we show that if momentum is due to continuation in the firm-specific component of returns, the profits of momentum strategies will decline at the rate of $\frac{1}{n}$, where n is the number of securities included in the portfolios, ultimately converging to zero in large portfolios. This pattern of declining profits is independent of the procedure used to form the portfolios. In contrast, we show that if momentum profits are due to cross-sectional variation in expected returns and risks of individual securities, the portfolio formation procedure is crucial to discriminating between the different sources of the profits. If portfolios are created randomly, the profits of momentum strategies will also decline at the rate of $\frac{1}{n}$. If, however, portfolios are formed on the basis of past return performance of individual securities, then expected momentum profits will show a dramatically different pattern. Under reasonable assumptions, the expected momentum profits will remain insensitive to the number of securities in the portfolios, especially for portfolios containing a small number of securities. The intuition for this result is quite simple: in the absence of observing the true mean returns and risks, the formation of portfolios based on the past performance of individual securities comes closest to forming portfolios based on the mean returns and risks themselves. This, in turn, implies that if there is cross-sectional variation in mean returns and risks of securities, this potential source of profitability will be maintained even when momentum strategies are implemented using portfolios of stocks. We can distinguish between, and determine the relative importance of, rational and irrational sources of momentum profits by using portfolios containing as few as two securities.

As noted above, attributing momentum profits to cross-sectional variation in mean returns and risks is intuitive and natural, and we are not the first to examine its role in explaining momentum strategy profits. Using a specific parametric model, Fama and French (1996) examine the performance of the Fama and French (1993) three-factor model in explaining a wide variety of patterns in security returns. The authors note that momentum is the main embarrassment to their three-factor model, in that loadings of stock returns on the factors fail to explain any part of the success of momentum strategies. Conrad and Kaul (1998) find that virtually all of the profits to momentum strategies can be traced to cross-sectional variation in unconditional mean returns. However, Jegadeesh and Titman (2002) dispute their findings, stating that while cross-sectional variation in unconditional mean returns is a legitimate theoretical candidate for the profits of momentum strategies, it has a trivial role to play in generating actual momentum profits.[3]

Given nearly fifteen years of evidence supporting a firm-specific explanation for momentum profits, why is there a need for another paper about cross-sectional determinants of momentum profits? Moreover, what is different about our approach? First, if continuation in firm-specific component of returns is responsible for the profits of momentum strategies, the implications for modern asset pricing are dire. An improper adjustment of asset prices to information that persists for three to twelve months is a blatant violation of market efficiency in its weakest form [Fama (1970)]. Even more importantly, idiosyncratic momentum rules out any cross-sectional variation in the mean returns and risks of individual securities which is another telling blow to modern finance. This conclusion is explicitly reached in Jegadeesh and Titman (2001, 2002), and Chen and Hong (2002) motivate their paper by a simple model of momentum in which "...every asset has the same mean and beta: $\mu_i = \mu$ and $\beta_i = \beta = 1$."

As alluded to earlier, these conclusions are based on empirical methodologies that are susceptible to erroneous inferences. Past research has felt compelled to propose and estimate a specific model for expected returns in order to determine the sources of momentum profits. This, however, creates the classic joint-inference problem emphasized by Fama (1970) of accepting/rejecting market efficiency or a specific model of expected returns. Moreover, the estimation of any model of expected returns is plagued by small-sample problems that could lead to erroneous inferences [see, for example, Black (1993) and Merton (1980)]. In the case of momentum, the joint-inference and estimation problems common to past studies could potentially have led to questionable conclusions about market efficiency and the intuitively appealing idea that risk and return should vary across different assets.

The difference in our approach relative to existing studies is that we develop sharp testable implications that can easily discriminate between rational and irrational sources of momentum profits regardless of a specific model of expected returns. Importantly, because our approach estimates neither a pricing model nor mean returns, it is immune to criticisms of existing approaches in the literature. That is, our results can be attributed neither to model misspecification nor to small sample errors in estimating means. Thus, we provide a powerful alternative methodology to standard decompositions of the profits of relative strength strategies.

Our findings are summarized as follows. Using the returns of all New York and American stock exchange stocks for 1926-2006 we show that, consistent with the existing literature, profits are strongly evidenced for relative strength trading strategies formed on the basis of individual firms.

[3] More recently, Ahn, Conrad, and Dittmar (2003) find that a nonparametric adjustment for risk explains much of the magnitude of momentum profits. Further, Bansal, Dittmar, and Lundblad (2005) show that cross-sectional variation in momentum portfolios' cash flow exposure to consumption risk explains virtually all of the variation in their average returns.

Based on the profitability of momentum strategies implemented using portfolios of securities we show that there is no evidence to support the widely held belief that momentum is an anomaly. When securities are combined into portfolios after being ranked based on their past performance, the profits of the momentum strategies do not decline with an increase in the number of securities. In fact, the average profit estimates of the portfolio strategies remain unchanged not only when the portfolios contain two securities, but even when reasonably large portfolios (with up to 50 securities in each) are used to implement the momentum strategy. This pattern is witnessed during the overall 1926-2006 period, and is present consistently in each of the four 20-year sub-periods. Consequently, there is no evidence of any momentum in the idiosyncratic component of individual-security returns. We also present detailed analysis and robustness tests to support this conclusion.

Although our evidence is strongly supportive of the notion that all of the momentum profits are driven by cross-sectional variation in required returns and risks of securities, some other sources of common variation across stocks could also generate these profits. For example, Moskowitz and Grinblatt (1999) and Lewellen (2002) consider scenarios in which common stock return movements give rise to momentum; the former suggest industry co-movements, while the latter proposes lead-lag relations among stocks of different (size, book-to-market, industry) characteristics. However, the industry effect has been criticized in Grundy and Martin (2001) because it is fragile and may be generated by market microstructure biases. They show that the profitability of the specific six-month strategies used by Moskowitz and Grinblatt (1999) disappears when there is a one-month gap between the ranking and holding periods of the strategies. Based on a battery of tests, they conclude that momentum is a firm-specific time series phenomenon. Chen and Hong (2002) provide evidence that lead-lag effects have no role to play in generating momentum profits. Indeed, the authors ask, "[w]hat are the economic mechanisms behind *individual stock* momentum?" (italics added) Our evidence suggests that this question is moot, as individual stock momentum (i.e., momentum driven by firm-specific news) does not exist.

The remainder of this paper is organized as follows. Section 2 contains the derivation of our theoretical predictions for patterns in momentum profits for individual-security-based strategies as well as portfolio-based strategies. This section also contains the formulation of hypothesis tests that can be devised to distinguish between, and determine the relative importance of, the competing (rational versus irrational) sources of momentum. In Section 3, we present the detailed results of these tests and some additional robustness experiments. Section 4 summarizes our main results and discusses their implications.

2 Sources of Momentum Profits

2.1 Strategy Using Individual Securities

Consider the momentum trading strategy implemented in the literature that ranks *individual securities* based on their relative strength. This strategy involves buying winners from the proceeds of selling losers, where winners and losers are determined based on the performance of a *single security* relative to the market. In this paper, we use all securities on the NYSE and AMEX, a fairly standard practice in the literature that permits us to analyze the profits of momentum strategies in a very intuitive and well-accepted manner.[4]

The weight received by each security in the momentum strategy, $w_{i\,t}$, is given by:

$$w_{i\,t} = \frac{1}{N}\left(r_{i,\,t-1} - r_{m\,,\,t-1}\right). \tag{1}$$

The equal-weighted market return is defined as $r_{m,t-1} \equiv \frac{1}{N}\sum\limits_{i=1}^{N} r_{i,t-1}$. Notice that the weight given to each security in this strategy is proportional to its deviation from the equal-weighted market return so that extreme winners/losers end up getting the most weight. A perusal of (1) also makes it clear that the sum of the weights add up to zero, thus making it a zero-investment strategy. The investment long (or short) is given by:

$$I_t = \frac{1}{2}\sum_{i=1}^{N}|w_{it}| = \frac{1}{2N}\sum_{i=1}^{N}|r_{i,\,t-1} - r_{m,\,t-1}|\,. \tag{2}$$

And the momentum profit can be written as:

$$\pi_t = \sum_{i=1}^{N} w_{it} r_{it}. \tag{3}$$

In our empirical results, we will primarily present average estimates of momentum profits given in (3), but we will also present the investment estimates and the scaled momentum profits, that is, profits scaled by the level of investment long/short. This is done primarily to provide a sense of the economic magnitude of the profits of the zero-investment strategy. In a world without wealth constraints, however, comparing scaled profits would make little sense because arbitrage profits can be scaled arbitrarily [see Lehmann (1990) and Lo and MacKinlay (1990)]. In this context, it is also important to realize that scaled profits could be higher for a strategy simply because the investment long/short is lower. Note that the investment in (2) is essentially a measure of the cross-sectional

[4] While most researchers have implemented such a strategy, some researchers have instead bought and sold extreme winners and losers [see, for example, Jegadeesh and Titman (1993)]. Regardless, the key issue is that the well-known and well-documented momentum strategy involves ranking *individual securities* based on past performance and then constructing a zero-investment portfolio.

dispersion of returns, and will be lower in some periods compared to others. It will also be lower for portfolios compared to individual securities. This property of scaled profits suggests that caution should be exercised in using it to make comparisons across different strategies.

To provide some minimal structure, suppose returns follow the familiar and simple one-factor model:

$$r_{it} = \mu_i + \beta_i f_t + \varepsilon_{i\,t}, \tag{4}$$

where μ_i is the unconditional expected return of security i, β_i measures the sensitivity of security's return to the single factor, f_t, and ε_{it} is the firm-specific or idiosyncratic component of security i's return. Clearly, the factor is assumed to have a zero mean for simplicity,

$$E(f_t) = 0. \tag{5}$$

Since the idiosyncratic component of a security's return is presumed by most to be the primary determinant of momentum profits, we will make the following clarifying assumptions about its behavior:

$$\begin{aligned} E(\varepsilon_{it}) &= 0; && (6)\\ E(\varepsilon_{it}\varepsilon_{j,\,t-k}) &= 0 \;\forall\; i \neq j \;\forall\; k; \text{ and} && (7)\\ E(f_t\varepsilon_{i,\,t-k}) &= 0 \;\forall\; k. && (8) \end{aligned}$$

The assumptions above are standard and, following the literature, we allow for momentum profits resulting from positive autocovariance in ε_{it}, i.e., $E(\varepsilon_{it},\, \varepsilon_{i,\,t-k}) \neq 0$. This setup allows for the presumed prevalence of under- (or continued over-) reaction of a stock's price to firm-specific events. As indicated in (7), however, the firm-specific components of any two securities are uncorrelated at all leads and lags because any correlation in the returns of two securities will result from the fact that both are affected by the common factor, f_t [see (8)]. This intuition can of course be generalized to a multifactor model. In other words, there cannot be any inter-temporal relation between the firm-specific components of the returns of any two securities in a well-specified asset-pricing model.

The model of returns in (4)-(8) is the simplest possible characterization of returns in that the mean returns and betas of securities are constants and common movements in security prices are captured parsimoniously by a single factor. While there is evidence that the means and betas of securities may be time-dependent and a multifactor model may be a more realistic characterization of returns, the essence of our theoretical predictions and empirical design can be conveyed using the simple model in (4)-(8). Another important reason for relying on this simple characterization of returns is that it forms the basis of most of the work on momentum strategies, including Jegadeesh and Titman (1993), and hence we are not departing from the well-accepted sources of momentum profits and their characterizations.

Algebraically manipulating (3) yields:

$$E(\pi_t) = \frac{1}{N}\sum_{i=1}^{N} E(r_{it}r_{i,\,t-1}) - E(r_{mt}r_{m,\,t-1}). \tag{9}$$

The decomposition in (9) shows that expected momentum profits have two components. The average product of the holding and ranking period returns of individual securities measures the average "continuation" in the returns of individual securities, while the product of the holding and ranking period market returns measures the expected continuation in the market. Specifically, the first is simply the average of the products of current and lagged individual security returns, while the second is the product of the returns of the market in the current and lagged periods. In other words, the momentum profits measure the continued relative strength averaged across individual securities. Since we will use this decomposition later in this paper, we will denote the first component as the average own-products of returns of individual securities used in the strategies, and the second component will be labeled the own-product of market returns.

It nevertheless is informative to use the return generating process in (4), and recombine the components of expected momentum profits given in (9) in a slightly different manner, to obtain a commonly used conceptualization of the sources of momentum profits [see Conrad and Kaul (1998), Jegadeesh and Titman (1993), Lehmann (1990), and Lo and MacKinlay (1990)]. Specifically, given the behavior of security returns described in (4)-(8), expected momentum profits can be decomposed as:

$$\begin{aligned} E(\pi_t) &= \left[\frac{1}{N}\sum_{i=1}^{N}\mu_i^2 + \frac{1}{N}\sum_{i=1}^{N}\beta_i^2 Cov(f_t, f_{t-1}) + \frac{1}{N}\sum_{i=1}^{N} Cov(\varepsilon_{it}, \varepsilon_{i,t-1})\right] \\ &\quad - \left[(\mu_m)^2 + (\beta_m)^2 Cov(f_t, f_{t-1}) + \frac{1}{N^2}\sum_{i=1}^{N} Cov(\varepsilon_{it}, \varepsilon_{i,t-1})\right] \\ &= \sigma_\mu^2 + \sigma_\beta^2 Cov(f_t, f_{t-1}) + \frac{N-1}{N^2}\sum_{i=1}^{N} Cov(\varepsilon_{it}, \varepsilon_{i,t-1}), \end{aligned} \tag{10}$$

where $\mu_m \equiv \frac{1}{N}\sum_{i=1}^{N}\mu_i$, $\beta_m \equiv \frac{1}{N}\sum_{i=1}^{N}\beta_i$, $\sigma_\mu^2 \equiv \frac{1}{N}\sum_{i=1}^{N}(\mu_i - \mu_m)^2$, and $\sigma_\beta^2 \equiv \frac{1}{N}\sum_{i=1}^{N}(\beta_i - \beta_m)^2$.

Equation (10) demonstrates why, for a given investment horizon, the momentum strategy is essentially a cross-sectional strategy – that is, in a world populated by rational investors, it is *expected* to produce positive profits. Specifically, the strategy should benefit from the cross-sectional variation in unconditional mean returns, σ_μ^2 , simply because it involves systematically buying (high-mean) winners financed from the sale of (low-mean) losers. In fact, as is obvious from (10) and as emphasized by Conrad and Kaul (1998), this will be the only source of momentum profits if asset prices follow random walks. The second potential source of momentum profits in (10) is also essentially a cross-sectional source. Although this source of profits will be economically important only if the common factor is positively serially correlated so that the strategy benefits from market timing, the profits will obtain only if there is cross-sectional variation in the betas of different securities.[5] It is only the third component of momentum profits that is related to any purely time-series behavior in returns. This source of profits is irrational because it is driven solely by the average positive serial covariance in the idiosyncratic components of the returns of individual securities. It is again important to emphasize that, following all previous papers, we assume that the firm-specific components of any two securities are uncorrelated at all leads and lags, i.e., $E(\varepsilon_{it}\varepsilon_{j,\,t-k}) = 0 \;\forall\; i \neq j$ and $\forall\; k$, because any correlation in the returns of the securities will result from the fact that both are affected by the common factor, f_t [see (8)].

As indicated in the introduction, several papers have come to the conclusion that the profitability of momentum profits is entirely due to serial covariance in the idiosyncratic component of returns, i.e., the irrational component [see, among others, Chen and Hong (2002), Grundy and Martin (2001), Jegadeesh and Titman (1993, 2001, 2002), Griffin, Ji, and Martin (2003), and Rouwenhorst (1998)]. An examination of (10) shows that this conclusion is disturbing because it implies that the market is not efficient. And this inefficiency is nontrivial because it takes several months for readily available public information to be incorporated in prices. Furthermore, this conclusion also suggests that the expected returns and betas of all securities are the same, thus bringing into question the foundation of asset pricing in a risk-averse world.

It is clear from (10) that we are confronted with an identification problem: to figure out the relative importance of the rational versus irrational sources of momentum profits, we have to specify and estimate a model for expected returns. This is because we can only observe the total realized profits of momentum strategies, and not the components. That is, we can only observe the sample analog of the left-hand side of (10), an equation that is widely used in the literature. To decompose profits using (10), however, we need to either know the "true" expected returns and risks of *individual securities*, or we should be able to estimate these parameters with precision. Unfortunately, we do not know the true mean returns and risks of securities. Further, they are difficult to measure because we do not have good models of expected returns and/or these models are notoriously difficult to estimate using limited data on relatively long-horizon returns.

Most papers that conclude that all the profits of momentum strategies arise from the serial covariance in the idiosyncratic component of security returns either estimate the mean returns and risks of securities or make an assumption about their cross-sectional variation. The most common approach is to estimate the betas of momentum strategies and, since the estimates are either zero or even marginally negative, these papers conclude that momentum has to be an anomaly. But betas are notoriously difficult to estimate and fragile [see Black (1993)], and there is a voluminous literature that estimated betas are not related to expected returns [see, for example, Fama and French (1992)]. There is a fair deal of consensus in the literature that returns are governed by

[5] The situation would be slightly different if we were to allow time-varying betas. A common factor and betas that are serially correlated with the same sign would strengthen the profitability of momentum strategies.

multiple factors but, even multifactor models such as the one proposed by Fama and French (1993), could be mis-specified and subject to estimation errors.

Several studies also investigate whether momentum strategies are profitable within different classes of stocks based, for example, on estimated betas and size [see, for example, Jegadeesh and Titman (1993, 2001)]. They find that these strategies too are profitable and again conclude that momentum therefore has to be an anomaly because cross-sectional variation in mean returns and betas is assumed to be negligible within these subgroups of securities. Such an assumption again cannot be accepted at face value without an explicit demonstration that estimated betas are strongly associated with the true risk of stocks and that firm size is strongly associated with the unconditional mean returns. Finally, while several researchers recognize that positive serial covariance in underlying factor(s) can yield momentum profits, they treat this potential source of profitability rather casually in their studies. The general tendency is to estimate the autocovariance or autocorrelation of medium-horizon returns of the equal-weighted market portfolio over short sample periods using non-overlapping data and then to dismiss this source of profits when the estimates are small positive/negative numbers [see Jegadeesh and Titman (1993) and Moskowitz and Grinblatt (1999)]. The problem here again is that these estimates are not only noisy but biased and negative even in randomly generated small samples precisely because the means have to be estimated [see Dixon (1944)].

In this paper, we develop an approach to determine the relative importance of the different sources of momentum profits that does not suffer from the problems intrinsic to past studies. Although we use (10) to intuitively understand the different sources of the profits of momentum strategies, we do not attempt to estimate the different components. We bypass the need to develop a model for, or to estimate, the required returns or risks of securities by deriving a set of empirical predictions that are invariant to these parameters. Specifically, we implement momentum strategies based on buying and selling *portfolios* of securities, and focus on the pattern of the momentum profits as increasingly more securities are included in the base portfolios. The key insight here is that the theoretical pattern of momentum profits driven by rational price behavior will be dramatically different from that driven by irrational price behavior. Therefore, it becomes empirically refutable whether or not the firm-specific news is the sole driver for the profitability of momentum strategies.

2.2 Strategies Using Portfolios of Securities

Consider momentum strategies that trade *portfolios* of stocks containing an increasingly large number of securities. As in the case of single-security strategy, we determine the relative strength of these portfolios by comparing their past performance to the market and implement a set of momentum strategies by buying winner *portfolios* and selling loser *portfolios* in proportion to their relative strength.

Specifically, consider a market with N stocks and a momentum strategy using portfolios, each of which consists of exactly n individual stocks. Without loss of generality, assume that N can be evenly divided by n so that the momentum strategy uses N/n base portfolios. We focus on the equal-weighting scheme for the base portfolio returns as well as the momentum portfolio return, so that:

$$w_{p,\,t} = \frac{1}{N/n}\left(r_{p,\,t-1} - \frac{1}{N}\sum_{i=1}^{N} r_{i,\,t-1}\right), \tag{11}$$

where $r_{p,\,t-1} \equiv \frac{1}{n}\sum_{i=1}^{n} r_{i,\,t-1}$.

The portfolio-based strategy in (11) is similar to the individual-security strategy reflected in (1); both are zero-cost strategies and both classify an asset into the winner or loser category based on the past performance relative to the market. There is a critical difference between the two, however. Equation (1) cannot be used to determine the relative importance of different sources of momentum profits without specifying and estimating a specific model of expected returns. The use of (11) with increasing number of securities in the base portfolios, however, allows us to bypass the need to specify a model of expected returns. We can determine the relative importance of the sources of momentum profits by instead focusing on the pattern of profits of portfolio strategies as the size of the base portfolios is increased.

To understand the benefits of using portfolios in implementing momentum strategies, it is useful to evaluate the expected profits to portfolio momentum strategies:

$$E(\pi_{pt}\,|_{n\geq 2}) = \frac{n}{N}\sum_{p=1}^{N/n} E(r_{pt}r_{p,\,t-1}) - E(r_{mt}r_{m,\,t-1}). \tag{12}$$

The expected profits of portfolio strategies are equal to the difference between the average expected continuation in the returns of the base portfolios and the expected continuation in the market returns or, equivalently, the difference between the own-products of the returns of the base and market portfolios. A comparison of (9) and (12) shows an identical second term in both, the expected continuation of the market returns, because we are always assessing the relative strength of either individual securities or portfolios with respect to the market. Consequently, we will focus on the pattern in the first term of (12), the expected average continuation in portfolio returns, relative to the first term in (9), the expected average continuation in individual-security returns.

There are two types of return products that affect the average expected continuation in the returns of the base portfolios: the own-products of the holding and ranking period returns of a specific stock, and the cross-products between the holding period return of one stock and the ranking period return of another stock. It is important to realize that the continuation in the returns of the base portfolios is affected by the cross-products of the returns of securities only within the same portfolios, not any pair of securities in the universe. As the number of securities in the base portfolios is increased, the own-products of individual security returns will contribute less to the momentum profits. At the same time, however, there will be a geometric increase in the cross-products within the base portfolios that will contribute to the momentum profits. Whether these cross-products strengthen or weaken the profitability of momentum strategies depends on how the base portfolios are formed in the first place, as there are potentially an infinite number of ways to do so.

We demonstrate below that a carefully designed method of grouping individual stocks into the base portfolios will generate distinctly different patterns in profits depending on whether rational or irrational price dynamics are the source of momentum. Specifically, we consider two ways of combining the stocks into portfolios. The first method is to form portfolios randomly. The second method is to combine securities by their rank ordering based on past performance, just as in the case of the traditional momentum strategy. In this new implementation, however, the n strongest winners (or losers) are grouped together into one base portfolio, the next n winners (or losers) in the order of their past relative strength into another, and so on. We show that the first method of forming portfolios randomly cannot help us distinguish between the rational and irrational sources of momentum profits because both sources lead to an identical pattern in profits with increasing n. Conversely, however, the second method of forming portfolios using the rank ordering of stocks based on their past performance leads to distinctly different patterns in profits with increasing n depending on the underlying source(s).

In order to devise a model-free method to determine the sources of profits, we need to examine the momentum in more detail. In the Appendix we show that expected profits to the individual-security and the portfolio-based momentum strategies can be written, respectively, as:

$$
\begin{aligned}
E(\pi_t) &= \left[\overline{\mu_i^2}^{ALL} - \overline{\mu_i \mu_j}^{ALL}\right] + \left[\overline{\beta_i^2}^{ALL} - \overline{\beta_i \beta_j}^{ALL}\right] Cov(f_t, f_{t-1}) \\
&\quad + \left[\overline{Cov\left(\varepsilon_{i,t}, \varepsilon_{i,t-1}\right)}^{ALL}\right], \text{ and} \qquad (13) \\
E(\pi_{pt}\,|_{n\geq 2}) &= \left[\frac{1}{n}\overline{\mu_i^2}^{ALL} + \frac{n-1}{n}\overline{\mu_i \mu_j}^{PORT} - \overline{\mu_i \mu_j}^{ALL}\right] \\
&\quad + \left[\frac{1}{n}\overline{\beta_i^2}^{ALL} + \frac{n-1}{n}\overline{\beta_i \beta_j}^{PORT} - \overline{\beta_i \beta_j}^{ALL}\right] Cov(f_t, f_{t-1}) \qquad (14) \\
&\quad + \left[\frac{1}{n}\overline{Cov\left(\varepsilon_{i,t}, \varepsilon_{i,t-1}\right)}^{ALL}\right],
\end{aligned}
$$

where the overline "——" denotes averages of the parameters, and the superscripts "ALL" and "$PORT$" denote the averages are for all securities in the universe and securities within base portfolios, respectively.[6]

The important common feature of (13) and (14) is that both profits can be decomposed into the same rational and irrational components. The first terms in both expressions denote profits from the cross-sectional variation in the unconditional mean returns, albeit of individual securities versus

[6] The expression for the expected profits of the single-security momentum strategy in (13) is nested in (14) for the special case of $n = 1$. We nevertheless provide explicit expressions for both the profits because it helps highlight the similarities and, more importantly, the differences between the two types of strategies.

portfolios. Similarly, the second terms denote any profitability due to cross-sectional variation in the betas of individual securities versus portfolios, which will be economically relevant only if the common factor, f_t, has any serial correlation. Finally, only the last terms of both (13) and (14) reflect the contributions of the irrational component caused by any mispricing due to firm-specific news.

The main difference between (13) and (14), however, is key to generating testable predictions that help distinguish between the rational versus irrational sources of momentum profits without requiring the specification and estimation of a model of expected returns. Note that the portfolio formation process, with an increasing number of securities in the portfolios, has very different implications for the first two versus the third components. The expected profits of single-security momentum strategies in (13) are determined entirely by the own-products and cross-products of *all securities* in the universe. On the other hand, the profitability of portfolio momentum strategies in (14) due to rational sources is increasingly affected by the weighted average cross-products of the unconditional means and the betas of all pairs of securities in the base portfolios. Specifically, the portfolio diversification process reduces the importance of the weighted own-products of both the unconditional means and the betas decline at the rate of $\frac{1}{n}\pi$ with an increase in the size of the base portfolios, but the importance of the corresponding cross-products increases. From (14) it therefore follows that the pattern in the profitability of portfolio momentum strategies using increasing number of securities will be determined entirely by the behavior of the weighted average cross-products of the means and betas of securities in the base portfolios, $\frac{n-1}{n}\overline{\mu_i\mu_j}^{PORT}$ and $\frac{n-1}{n}\overline{\beta_i\beta_j}^{PORT}$, respectively, relative to the average cross-products of all securities in the universe, $\overline{\mu_i\mu_j}^{ALL}$ and $\overline{\beta_i\beta_j}^{ALL}$. If we can combine securities with similar mean returns and betas into each base portfolio, the profitability of the portfolio momentum strategies could remain invariant to the size of the portfolios.

Conversely, and importantly, the portfolio formation process will have a starkly different effect on any profits generated by the mispricing of securities caused by delayed reactions or continued overreactions to firm-specific news. Similar to the effects of the portfolio diversification process on the importance of both the own-products of the unconditional means and betas, the own-product of the idiosyncratic component of returns also declines at the rate of $\frac{1}{n}$ with the size of the base portfolios. There however is no compensating effect due to cross-products because firm-specific components of any two securities cannot be correlated with each other at any leads or lags, i.e., $E(\varepsilon_{it}\varepsilon_{j,t-k}) = 0 \ \forall \ i \neq j$ and $\forall \ k$. Consequently, the profits of portfolio momentum strategies will shrink at the speed of $\frac{1}{n}$,regardless of the manner in which individual securities are combined into base portfolios.

As mentioned above, we use two methods to combine securities into portfolios. The first method combines securities randomly into base portfolios. From (14) and the above discussion it is clear that any contribution to the profitability of momentum strategies due to continuation in the idiosyncratic components of stock returns will decline at the rate of $\frac{1}{n}$, where n is the number of securities included in the portfolio. It however turns out that the average cross-products within base portfolios will converge to the average cross-products of all securities in the universe, that is, $\overline{\mu_i\mu_j}^{PORT} = \overline{\mu_i\mu_j}^{ALL}$ and $\overline{\beta_i\beta_j}^{PORT} = \overline{\beta_i\beta_j}^{ALL}$, thus leading to decline at the rate of $\frac{1}{n}$ in any profits generated by rational sources as well. Therefore,

$$E(\pi_{pt}\,|_{n\geq 2}) = \frac{1}{n}E(\pi_t\,|_{n=1}). \tag{15}$$

In other words, the method of forming portfolios randomly fails to provide any testable predictions that enable us to distinguish between rational and irrational sources of profits. This occurs because expected momentum profits will decline at the rate of $\frac{1}{n}$ regardless of whether the rational or irrational causes are at play. This method nevertheless serves as an important robustness check of our methodology.

Our second method of forming the base portfolios first ranks securities based on their individual past performance, and then combines them into portfolios before classifying them as winner and loser portfolios relative to the market. When securities with adjacent ranks of prior performance are grouped into the same base portfolio, the average cross products within base portfolios will converge to the average own-products of all securities in the universe, i.e., $\overline{\mu_i\mu_j}^{PORT} = \overline{\mu_i^2}^{ALL}$ and $\overline{\beta_i\beta_j}^{PORT} = \overline{\beta_i^2}^{ALL}$. This occurs because, in the absence of observing the true unconditional mean returns and risks, the ranking of stocks based on past returns will on average mimic their ranking based on the unobservable unconditional mean returns and risks. This is likely to be the case especially for momentum strategies with portfolios containing a small number of stocks because a smaller set of adjacent stocks ranked based on their past performance are likely to have similar mean returns and risks. Hence, the expected momentum profits from using portfolios with such features can be written as:

$$\begin{aligned} E(\pi_{pt}\,|_{n\geq 2}) \;&=\; \left[\overline{\mu_i^2}^{ALL} - \overline{\mu_i\mu_j}^{ALL}\right] + \left[\overline{\beta_i^2}^{ALL} - \overline{\beta_i\beta_j}^{ALL}\right] Cov(f_t,\, f_{t-1}) \\ &\quad + \left[\frac{1}{n}\overline{Cov\left(\varepsilon_{i,t}, \varepsilon_{i,t-1}\right)}^{ALL}\right]. \end{aligned} \tag{16}$$

The formation of portfolios using the rank ordering of individual securities based on their past relative strength has two interesting consequences: there is no "diversification effect" for small n for the "systematic" component of momentum profits, whereas the idiosyncratic component of momentum profits is reduced at the rate of $\frac{1}{n}$. As n is increased, stocks with increasingly different mean returns and betas will be grouped together and thus the cross-products will tend to become smaller relative to the own-products, leading to a decline in the average cross-products, $\overline{\mu_i\mu_j}^{PORT}$ and $\overline{\beta_i\beta_j}^{PORT}$. This decline will initially be balanced by a larger weight of $\frac{n-1}{n}$. Even when n becomes very large, however, the expected profits will not converge to zero because, if there is any cross-sectional dispersion in mean returns and betas of individual securities, $\overline{\mu_i\mu_j}^{PORT} > \overline{\mu_i\mu_j}^{ALL}$

and $\overline{\beta_i\beta_j}^{PORT} > \overline{\beta_i\beta_j}^{ALL}$ [see (14)].[7]

The methodology of implementing portfolio momentum strategies using the rank ordering of securities based on their past relative strength has some appealing properties. First, this design provides very distinct testable predictions to distinguish between the different sources of momentum profits. If, as most studies have concluded, irrational price movements are the sole (or predominant) cause of momentum, the profits of these strategies will decline at the rate of $\frac{1}{n}$ to zero when portfolios of increasing number of securities are used to implement momentum strategies. [That is, $\mu_i = \mu_j$ and $\beta_i = \beta_j$ for all $i \neq j$ and only the third term of (16) matters.] Conversely, however, if cross-sectional variation in expected returns and risks is the primary source of momentum, these strategies may witness profits that remain unaltered as the number of securities in the portfolios is increased. [That is, the third term of (16) drops out entirely.] Although profits will decrease as the number of securities in the portfolios, n , becomes very large, they will not converge to zero. The most appealing aspect of these portfolio strategies is that we do not need to form large portfolios to be able to distinguish between the rational and irrational components *and* determine their relative importance. In fact, the sharpest hypotheses tests obtain for portfolios containing a small number of securities. For example, with only two securities in a portfolio, if irrational continuation in the idiosyncratic component of returns is the only source of profits, the expected momentum profits will be $\frac{1}{2}$ the profits of the traditional single-security momentum strategy. Conversely, there is likely to be no decline in the profitability of the strategy if rational cross-sectional variation in mean returns and risks of individual securities is the only source(s). This follows because two securities ranked based on their past performance are, on average, likely to have very similar mean returns and risks.

The second aspect of the empirical design is that it involves examining the pattern of momentum profits with respect to the portfolio size, without conducting any decomposition of momentum profits. All that is required is the implementation of a set of portfolio-based momentum strategies. These portfolios need to have increasing number of securities, but they are formed using the same set of stocks that are used in the commonly-implemented single-security momentum strategy. Our tests therefore effectively bypass the need to specify and estimate a model of expected returns. This aspect of our tests is not only novel, but it will also help us reach conclusions that are unaffected by estimation errors and biases present in previous studies.

Finally, although the autocovariance in the common factor, f_t, has the potential of altering the level of expected momentum profits for all n, the empirical verification of the profit pattern with increasing n does not depend upon the magnitude or the sign of the autocovariance of the factor. We cannot determine whether time-series correlation in the factor contributes to the rational component, but we can, with precision, determine whether irrational continuation in the idiosyncratic

[7] To see why these inequalities hold, recall that our method of forming portfolios on the basis of ranking period returns ensures that there are no cross-products of returns between a winner and a loser within the same base portfolio, whereas the cross-products of returns for all securities in the entire population will necessarily include cross-products of returns of winners and losers. The momentum profits will of course decline to zero if the base portfolios are formed with an increasing number of randomly selected securities.

components of individual security returns contributes to momentum profits.

3 Empirical Evidence

In implementing the momentum strategies, we include all stocks on the New York and American stock exchanges (NYSE/AMEX) between 1926 and 2006 and, following Jegadeesh and Titman (1993), we focus on the six-month investment horizon. Monthly returns of individual stocks are first compounded over a six-month ranking period, and then compared to a benchmark return (either equal-weighted or value-weighted market return) so as to separate winners from losers.[8] The proceeds from selling losers are used to buy winners, with the extreme winners and losers receiving highest weights [see (1) for the equal-weighted strategy]. The resulting portfolio is held for the subsequent six-month holding period, and the momentum profit is computed based on the compounded returns for individual stocks during the holding period. If a stock drops out of our sample during the holding period, we include its compounded return for the months that it survives in the holding period. This procedure is repeated on a rolling basis for the entire sample period, and the time stamp of momentum profits ranges from July, 1926 to June, 2006.

We maintain a one-month gap between the ranking and holding periods in order to mitigate the effects of market microstructure biases that have been frequently emphasized in the literature. We also repeat the entire exercise without skipping one month, and the pattern of results is qualitatively similar. Although there is attenuation in the profitability of the strategies conducted over the entire sample period, consistent with the results in Jegadeesh and Titman (1993) this attenuation is driven by the 1926-1946 period. The profitability of the strategies remains very similar during all other sub-periods.

In this paper, the momentum strategies are executed based using individual stocks and portfolios. For the *single-security* strategy, each winner/loser consists of only one stock, and the description above suffices. For *portfolio-based* strategies, each winner/loser is a portfolio of n stocks that is formed based on certain attributes during the ranking period. We consequently also need to compute the portfolio returns using the compounded individual stock returns during the ranking and holding periods. The relative performance of the portfolio returns during the ranking period affects both the determination of winners versus losers and the weights for the momentum strategies. In both the single-security and portfolio strategies, the benchmark return for the ranking period is calculated from the compounded returns of all the individual stocks in the population.

[8] We include a security in the sample as long as it has non-missing returns for at least one month in the ranking period. Requiring securities to have monthly returns for all six months reduces the number of securities in our sample. Regardless, our results remain qualitatively unchanged even when we impose this requirement.

3.1 The Robustness and Consistency of Momentum Profits

Table 1 contains estimates of average profits of the traditional momentum strategies that use individual securities. Panel A contains the estimated profits for the standard equal-weighted strategies implemented in past research, and discussed in detail in Section 2. We report the average estimates for the overall 1926-2006 period and four 20-year sub-periods within. The heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses [see Davidson and MacKinnon (1993) and Newey and West (1987)]. In these strategies, securities are ranked in the previous six-month period based on their performance relative to the equal-weighted market. The strategy then weights these deviations from the market equally, creating a zero-investment portfolio. The average estimate for the profit in the overall period, 1926-2006, is statistically significant at 0.417×10^{-2}. Since this is a "return" on a zero-investment strategy, we also provide estimates of the investment long/short in the strategy that, on average, amounts to \$0.094. The scaled profit per dollar investment long/short is 4.38% over a six-month period, which translates to an annualized return close to 9%.

This estimate is similar to the estimates reported in the literature, although most studies implement the momentum strategies over the post-1962 period. Not surprisingly therefore the sub-period estimates of the average profits are also consistently statistically and economically significant. The point estimates range between 0.284×10^{-2} during 1946-1966 to 0.549×10^{-2} during the most recent 20-year period. Since there is variation in the level of investment long/short, the return per dollar long also varies between 2.82% and 5.48%. These estimates not only verify the consistency and robustness of the momentum strategy over an 80-year period, they also translate into substantial annualized returns between 5.7% and 11.3%. And, interestingly, the most profitable period spans the 20 years after the discovery of the strategy by Jegadeesh and Titman (1993).

Given the robustness of the momentum strategy over time, it appears natural to also test its robustness across different types of stocks. This is especially important because Jegadeesh and Titman (1993, 2001) claim that the strategy is consistently profitable across stocks of different market values. They arrive at this conclusion by implementing the strategy *within* market-value-based categories of stocks. This however is not the same as evaluating the importance of different types of stocks in determining the profitability of the momentum strategy applied to all stocks. To gauge whether firms of different size play a different role in determining momentum profits, we instead conduct a simple test. We implement a value-weighted momentum strategy with the weight in (1) is replaced by

$$w_{i\,t} = v_{i,\,t-1}\left(r_{i,\,t-1} - r_{m,\,t-1}\right), \tag{17}$$

where $v_{i,\,t-1}$ is the relative market-value of security i at time t-1, and $r_{m,\,t-1}$ is the value-weighted market return.[9] This strategy is also a zero-investment strategy, yet it allows us to evaluate the

[9] The relative market-value $v_{i,\,t-1}$ is based on the most recent market value of equity in the ranking period. It is also used for computing the holding period returns for portfolio-based strategies (to be discussed shortly) as we

importance of different types of firms in determining momentum profits.

The average estimates of the profitability of the value-weighted momentum strategy are provided for the overall period and the four sub-periods in Panel B of Table 1, with the heteroskedasticity and autocorrelation consistent t-statistics in parentheses. The most immediate and noteworthy aspect of the profit estimates is that they are dramatically lower than the estimates for the equal-weighted strategies reported in Panel A and all past studies. For the overall period, the average estimate of 0.192×10^{-2} is less than 50% of the corresponding estimate of 0.417×10^{-2} in Panel A. The sub-period estimates are even more telling in that the strongest two sub-periods of 1966-1986 and 1986-2006 for the equal-weighted strategies are the weakest ones for the value-weighted ones. Specifically, the estimate for 1966-1986 in Panel B that forms the main part of the original study by Jegadeesh and Titman (1993) is only 35% of the corresponding estimate in Panel A. Similarly, the value-weighted strategy nets a profit of only 0.157×10^{-2} during the latest 1986-2006 period, which is less than 30% of the equal-weighted profits of 0.549×10^{-2}. The t-statistic of this average estimate also drops sufficiently to bring into question its statistical significance.

The evidence in Panel B also shows how scaled momentum profits, though useful to evaluate the economic importance of a specific strategy, can convey a misleading impression when used to compare the profitability of different strategies. Though the estimates of the average scaled profits in Panel B are also substantially lower than their equal-weighted counterparts in Panel A, they are inflated by the significantly lower investment estimates. The investment estimates are predictably lower because value-weighted returns should have lower cross-sectional dispersion.

The estimates in Table 1, Panel B, convey a simple and obvious message: firm-size has a major role to play in the momentum phenomenon. This evidence is contrary to the evidence in Jegadeesh and Titman (1993, 2001), but is consistent with the findings of Hong, Lim, and Stein (2000) who argue that momentum is related to firm-size when the smallest-sized firms are excluded from the strategies. We believe that the evidence in Table 1, Panel B, though admittedly preliminary, is suggestive of a rational explanation for the success of equal-weighted momentum strategies implemented in the literature. Note that the decomposition of the expected momentum profits in (10) shows that cross-sectional variation in required returns and risks will contribute to these profits. The expected momentum profits of the value-weighted strategy should therefore be substantially lower than the commonly implemented equal-weighted strategies because the value-weighted cross-sectional variation in mean returns and betas should be lower. This is because small firms that tend to have higher average returns and risks get much lower weights in this strategy. The results in Table 1, Panel B, are however also consistent with the hypothesis that momentum results from the underreaction of the prices of small firms with sparse analyst following to firm-specific news [see, for example, Hong and Stein (1999) and Hong, Lim, and Stein (2000) among several others]. The results for more direct tests of these alternative hypotheses, described in Section 2 above, are now presented in the following sections.

assume that there is no rebalancing during the holding period.

3.2 Expected Returns or Firm-Specific Under- (or Continued Over-) reaction?

To distinguish between the rational versus irrational/behavioral sources of momentum, we conduct the portfolio-based tests outlined in Section 2. We first present evidence for portfolio strategies that combine securities rank ordered based on their past performance because they provide distinctly different predictions for the two sources of profits. Evidence based on the random portfolio strategies are then presented as part of robustness checks of our methodology. Specifically, individual stocks that have already been categorized into winners and losers based on their relative performance during the ranking period are now combined into base portfolios of size n by sequentially picking off n stocks at a time from each category. The strongest n winner stocks are grouped in the first winner portfolio; the next n winner stocks are grouped in the next winner portfolio; repeating this procedure until all winner stocks are part of some winner portfolio. Similarly, the strongest n loser stocks are grouped in one loser portfolio; the next n loser stocks are grouped in another loser portfolio; and so on.[10]

Table 2, Panel A, contains estimates of the profitability of *portfolio* momentum strategies conducted using the same set of ranked securities that are used in the *single-security* strategy reported in Table 1. We also report the heteroskedasticity and autocorrelation consistent t-statistics. From the detailed analysis presented in Section 2, recall that the two alternative sources of momentum profits have distinctly different implications for the profitability of these portfolio strategies. If firm-specific news leads to momentum in returns, the average profits of equal-weighted portfolio strategies should decline at the rate of $\frac{1}{n}$, and eventually to zero, as the number of securities included in the portfolios (n) is increased. Conversely, however, if the ranking of securities signals differences in their expected returns and risks, the profits of the portfolio strategies are likely to remain the same for small n . And even for very large n , the profits should remain economically and statistically greater than zero. For purposes of our tests, we choose portfolios of sizes ranging between two and 50. Although we can test the predictions of our tests with just a few securities in a portfolio, for completeness we include as many as 50 securities in the portfolios. By limiting the maximum size of portfolios to 50 securities, however, we ensure that a reasonable number of winner/loser portfolios can be formed for implementing the trading strategies.

The estimates of the average profitability of the portfolio strategies reported in Table 2, Panel A, provide compelling evidence in favor of a rational story for the success of momentum strategies. There is no evidence in support of the widely held belief that momentum is an anomaly driven solely (or predominantly) by the under- (or continued over-) reaction to firm-specific news. For the overall period, the average estimates of the equal-weighted momentum profits do not decrease with

[10] Note that the last winner/loser portfolios may contain less than n stocks, and the stocks in these two portfolios have returns fairly close to the benchmark return. Our results remain unchanged if we exclude these stocks from our strategies by requiring all base portfolios to include exactly n stocks during the ranking period. Base portfolios with unbalanced size can also occur when stocks that are present in the ranking period drop out of the sample during the holding period. We find that requiring exactly n stocks in both the ranking and holding periods also does not alter our results.

n. In fact, the estimates for all values of n are statistically indistinguishable from 0.417×10^{-2}, the average profit for the overall period reported in Table 1, Panel A (and also in the first row of Table 2, Panel A). One simple, yet powerful, way to evaluate the implications of the evidence is to consider the profitability of the portfolio with only two securities (i.e., $n = 2$) because the predictions of the two alternative hypotheses are starkly different. If irrational price continuation is the only source of profits the average profits for $n = 2$ should be $\frac{1}{2}$ of the profits of the strategy implemented with individual securities (i.e., $n = 1$). In contrast, there should be no change in the average profits if the profits are driven by cross-sectional variation in required returns and risks of securities. The evidence strongly supports the rational hypothesis because there is no attenuation in average profits; in fact the point estimate of average profits is slightly higher at 0.436×10^{-2} as compared to 0.417×10^{-2}. The theory in Section 2 [see (16)] predicts no attenuation in momentum profits of strategies trading portfolios of two ranked securities only if there is no firm-specific price continuation *and* if adjacent pairs of ranked securities have the same mean returns and risks. Consequently, our evidence suggests that there can be no irrational momentum in the firm-specific component of individual-security returns. The evidence is instead consistent with momentum being driven entirely by cross-sectional variation in mean returns and risks of securities.

The results for the four sub-periods provide very similar conclusions. There is a slight decline in average profits for portfolios containing more than 20 securities only in the 1926-1946 period, but even in this sub-period the profitability of the portfolio strategies are statistically indistinguishable from the corresponding single-security strategies for all values of n. Importantly, and as in the overall period, the point estimates of the average momentum profits for $n = 2$ are slightly higher than $n = 1$ for all but the 1926-1946 sub-period. This again suggests that there is no evidence of momentum in the firm-specific component of individual-security returns.

An additional appealing aspect of momentum strategies that use portfolios combining securities based on their past ranked returns is that the investment estimates provided in Table 2, Panel B, are identical across the different-sized portfolios within each period. This is not surprising because we always combine winners with other winners, and losers with other losers, in forming the portfolios. Thus the expected value of the investment, see (2), has to be identical for all portfolios within a particular period. This property of the investment outlays has the advantage of allowing us to use the scaled momentum profits to test whether momentum is a rational or an idiosyncratic phenomenon. Panel C again shows that under- (or continued over-) reaction to idiosyncratic news cannot have any role in generating momentum profits. Consistent with cross-sectional variation in mean returns and risks of securities, the average scaled profit estimates stay statistically unchanged with an increase in the number of securities included in the portfolios. There is a very slow decline in scaled profits during 1926-1946, but the profits of the largest portfolio (with $n = 50$) are still statistically indistinguishable from the scaled profits of the traditional momentum strategies. In any event, even the point estimate average scaled profits for $n = 50$ is more than 75% of the scaled profits of the single-security strategy, which is much higher than the 2% predicted by the irrational hypothesis.

Although the precise predictions of our tests presented in Section 2 are directly applicable to equal-weighted portfolio strategies presented in Table 2, for completeness we also execute value-weighted portfolio strategies. Both average estimates and the heteroskedasticity and autocorrelation consistent t-statistics for these strategies are reported in Table 3. The evidence again provides strong support for the rational explanation of the momentum phenomenon. In stark contrast to the prediction of the irrational hypothesis for momentum, the average profits remain statistically unchanged when an increasingly large number of securities are included in the base portfolios. In fact, even during 1926-1946 there is no attenuation in the profitability of the momentum strategy even for large n, a pattern witnessed in estimates of both the average profits and the scaled average profits.

Finally, as mentioned earlier, all the tests reported in Tables 2 and 3 maintain a one-month gap between the ranking and holding six-month periods to attenuate any market microstructure biases. We have also repeated the entire battery of tests without skipping one month, and the pattern of our results is qualitatively similar. This is in contrast to the Moskowitz and Grinblatt (1999) industry explanation for the success of the six-month momentum strategy, which is quite sensitive to the potential microstructure biases present in the data [see Grundy and Martin (2001)]. This suggests that common factor(s) unrelated to industry affiliation of securities appear to be the main cause of the success of momentum strategies.

We believe that the major advantage of the tests presented in Tables 2 and 3 is that they do not depend on a specific model of expected returns and, more importantly, they do not call for the estimation of the components of momentum profits. As mentioned earlier, in an attempt to determine the profit sources of momentum strategies all past studies have instead attempted to estimate decompositions similar to the one in (10). Unfortunately, apart from the fact that we do not have a theoretically-grounded model of expected returns, estimation of even unconditional means, covariances, and variances are all subject to small-sample biases [see, for example, Black (1993) and Merton (1980)].

We believe that the results in Tables 2 and 3 are compelling on their own but, for completeness, in the sections that follow we present a more detailed analysis of the profits of the portfolio strategies. We also provide some robustness checks to convince the skeptical reader that there is no momentum arising from the firm specific components of returns.

3.3 A Detailed Analysis of Momentum Profits

We show in Section 2 that momentum strategies trading portfolios of securities that are grouped based on the rank ordering of their past performance will remain profitable if cross-sectional variation in mean returns and risks is the main source of profits. This prediction is confirmed by the average profit estimates presented in Tables 2 and 3. In this section, we present the individual-security return dynamics underlying this pattern to show that momentum cannot be driven by continuation in the firm-specific component of returns. The dynamics is instead consistent only with cross-sectional variation in required returns and risks of securities. Specifically, and as shown theoretically in Section 2, the average profits of portfolio momentum strategies will remain unchanged because the declining profit contribution by the own-products of individual securities will be offset by the additional profit attributed to the cross-products within the base portfolios [see (13) and (14)]. If the ranking of past returns reflects differences in mean returns and risks, and at least some stocks have similar mean returns and risks, the average profits will remain largely unchanged for small n. Even as n becomes large, the cross-products of the securities in the base portfolios remain larger than the own-product of the market portfolio which, in turn, reflects the average cross-products of the returns of the entire population of securities. If the factor is serially uncorrelated, from (14) it implies that $\overline{\mu_i\mu_j}^{PORT} > \overline{\mu_i\mu_j}^{ALL}$.

To evaluate the sources of momentum profits, in Table 4, Panels A and B, we present the weighted average own- and cross-products of the returns of securities *within* the base portfolios that are used to implement the equal-weighted strategies. Since the un-weighted average cross-products *within* the base portfolios play an important role in distinguishing between the distinctly different dynamics of average profits in a rational versus irrational world, we report them in Panel C. These are simple transformations of the weighted average cross-products reported in Panel B [see (14)], and the two series converge for large n. Panel D contains average estimates of the own-product of the market return, which reflect the average cross-products of all securities in the universe. Note that, unlike past studies, we do not decompose profits as in (10). Although such an exercise would increase our understanding of the sources of momentum profits, it will be plagued by model misspecification and estimation errors, thus rendering any inferences unreliable. We instead analyze own- and cross-products of security returns because they do not require the estimation of mean returns and hence do not suffer from small-sample biases [see first parts of (13) and (14)]. Table 4 also contains the heteroskedasticity and autocorrelation consistent t-statistics for all the estimates.

Before we analyze the evidence in Table 4, it may help to explain the various estimates, and their relation to average profits. From (14), and focusing for convenience only on the first bracket, average profits of any strategy will have three components: the average estimate of the own products within a base portfolio ($\frac{1}{n}\overline{\mu_i^2}^{ALL}$, see Panel A) *plus* the average estimate of weighted cross-products within the base portfolios ($\frac{n-1}{n}\overline{\mu_i\mu_j}^{PORT}$, see Panel B) *less* the average estimate of the cross-products of all securities in the population ($\overline{\mu_i\mu_j}^{ALL}$, see Panel C). For the traditional momentum strategy that classifies individual securities into winners and losers, that is, for $n = 1$, the average profit of 0.417×10^{-2} for the overall period (see Table 1, Panel A) is the average estimate of the own-products, 1.239×10^{-2}, reported in the first row of Table 4, Panel A, less the average own-product of the market, 0.810×10^{-2}, shown in Panel D.[11] Note that for this strategy by construction there are no cross-products within the portfolio, and hence all estimates in the first row of Panels B and C are zero.

To gauge whether the estimates in Table 4 are consistent with a rational explanation, it is again instructive to evaluate the evidence for $n = 2$, the portfolio strategy with the fewest number of securities in each portfolio, and compare it to the traditional momentum strategy that evaluates individual securities. For $n = 2$, the average of the own-products of individual security returns is reduced by 50% to 0.620×10^{-2} (see second row of Panel A), but this reduction is compensated fully by the weighted average cross-products among the securities in the portfolios, 0.653×10^{-2} (see first row of Panel B). The most important aspect of the results in Table 4 is that the *difference* between the sum of the weighted average own- and cross-products of the securities within the base portfolios and the weighted average cross-products for all securities in the universe is maintained, that is, [(Panel A + Panel B) - Panel D] remains unchanged. Thus, as predicted in Section 2 and consistent with rational cross-sectional dispersion with required returns and risks of securities, the average profits remain unchanged for the overall period.

The estimates for strategies using portfolios with two securities is not only inconsistent with the irrational explanation widely accepted in the profession, but it also shows that price continuation due to firm-specific news has no role to play in determining momentum profits. For $n = 2$, both the rational and the irrational hypotheses predict that the average estimate of the own products of returns in the portfolios will drop by 50% to 0.620×10^{-2}. The estimates in Panel A alone therefore cannot help us distinguish between the rational and irrational explanations unless, like previous studies, we specify and estimate a model for the required returns of individual securities to enable us to attribute the 50% drop to either the rational or irrational sources.

The methodology introduced in this paper bypasses this potentially slippery step, yet helps us distinguish between, and determine the relative importance of, the rational versus irrational explanations. The key to our ability to do this lies in the behavior of the *un-weighted* average cross-products *within* the base portfolios, reported in Table 4, Panel C, *relative* to the average cross-products of all securities in the universe shown in Panel D.[12] If momentum is caused by under- (or over-) reaction to firm-specific news, the un-weighted average of the cross-products within the base portfolios reported in Panel C of Table 4 will be equal to the average cross-products of all securities in the universe in Panel D for all n, including $n = 2$. This will obtain because the un-weighted average cross-products within the base portfolios cannot be affected by any price continuations due to firm-specific news and any cross-sectional variation in mean returns is ruled out by construction [see (15)]. Inconsistent with the irrational hypothesis, the un-weighted average cross-products

[11] The minor difference is the result of the fact that some securities in the ranking period drop out of the holding period which, in turn, affects the estimates presented in Table 4.

[12] Each base portfolio consists of n stocks and thus produces $n(n-1)$ cross-products of returns. Taking the average of all such cross-products across all the N/n base portfolios, we get the measure reported in Panel C of Table 4. This measure is called un-weighted because once we multiply it by the $(n-1)/n$ factor (i.e., the weight) we arrive at the weighted average cross-products that are reported in Panel B of Table 4. See (14).

within the base portfolios containing only two securities is 1.306×10^{-2}, which is much higher than the average cross-products of all securities in the universe, 0.810×10^{-2}. Importantly, the un-weighted average cross-products are very similar in magnitude to (and, in fact, slightly higher than) the average own-products, thus ruling out the possibility of any price continuations due to firm-specific news. Equivalently, this evidence suggests that cross-sectional variation in required returns and risks of securities are solely responsible for generating momentum profits.

As the number of securities in the portfolios of winners/losers increases, the own-products of the securities become less important, while the cross-products play an increasing role in contributing to the momentum profits. Even for $n = 50$, the average of the weighted and un-weighted cross-products (which are quite similar at 1.222×10^{-2} and 1.247×10^{-2}) remain very close to the average of the own-products of the individual securities, 1.239×10^{-2}, and thus substantially greater than the estimated own-product of the market return of 0.810×10^{-2}. Thus, it is not surprising that the average profitability of the portfolio momentum strategies remains unchanged even with a fairly large number of securities in each portfolio. This again is inconsistent with momentum being driven, to any measurable degree, by under- (or continued over-) reaction of stock prices to firm-specific news.

As can be expected, the cross-sectional variation in expected returns and risks are likely to vary over time. Consequently, it is not surprising that the profitability of momentum strategies reported in Table 1, Panel A, varies over the different sub-periods. In each of the four sub-periods, however, the behavior of the average own- and cross-products within the base portfolios mimics the pattern in the overall period. Specifically, when there are a few securities in each portfolio, the systematic decline in the importance of own-products is fully compensated by the cross-products within base portfolios, thus maintaining the profitability of the momentum strategies. This occurs because for small n the un-weighted average cross-products within the base portfolios (see Panel C) are much larger than the average cross-products for the entire universe of securities (see Panel D). And even when the portfolios contain a fairly large number of securities, the average estimates of the weighted and un-weighted cross-products remain economically and statistically greater than estimates of the own-products of the market (compare the last row of Panel B with C to Panel D). Again it is the difference between these two average estimates that ensures the profitability of the momentum strategies that contain even 50 securities in each portfolio, though it is important to remember that these securities are ranked based on their past performance before they are combined into portfolios. Hence, there is no evidence of irrational behavior in stock prices in any sub-period.

Before we present a final set of robustness tests to confirm our conclusions, a few comments about our methodology relative to past studies are in order. Previous studies do not analyze cross-products, but instead attempt to estimate serial covariances and correlations of the factor(s). For example, studies find that the serial covariance and correlation of the equal-weighted market are small negative numbers in the post-war period [see, for example, Jegadeesh and Titman (1993), Lewellen (2002), and Moskowitz and Grinblatt (1999)]. As our analysis in Sections 2 and 3 hopefully establishes, in order to determine whether momentum is an anomaly caused by under- (or continued over-) reaction to firm-specific news, there is no need to determine whether the factor is positively autocorrelated or not. This follows because any such autocorrelation will matter only if the risks of different securities are measurably different, and any such cross-sectional variance in risks is a rational phenomenon. In any event, unlike own- and cross-products, estimates of serial covariances and correlations of returns are biased in small samples because mean returns need to be estimated.

Although we will refrain from estimating the means, covariances, and autocovariances of even the return on the market portfolio, it may nevertheless be informative to get an idea of whether the time-series dynamics of factor(s) contribute to the profitability of momentum strategies. This is only important because the cross-sectional variation in unconditional mean returns always contributes to the profitability of momentum strategies, while the autocovariance in the factor(s) may exaggerate or attenuate the profits depending on whether it is positive or negative [see the decompositions in (13) and (14)]. At least for the overall period, where we have a reasonably large sample of overlapping six-month returns, our average estimate of the market's own-product suggests that the factor(s) are likely to have contributed to the profitability of momentum strategies. Specifically, the average estimate of the market's own-product is 0.810, which would imply a positive serial covariance in the factor unless the annualized mean return for the market was greater than 18% during 1926-2006![13]

3.4 A Robustness Check

Tables 2, 3, and 4 contain compelling evidence that cross-sectional variation in expected returns and risks is the dominant source of the profitability of momentum strategies. The tables confirm the prediction developed in Section 2 that if past relative performance is indicative of differences in mean returns and risks, momentum strategies trading portfolios of past winners and losers should continue to remain profitable. The tables lend no support for the notion that momentum profits are solely (or predominantly) attributable to firm-specific news. The evidence is inconsistent with the prediction that if past performance is a signal of future continuation in firm-specific returns, the profits of these portfolio strategies should decline at the rate of $\frac{1}{n}$ to zero with an increase in the size of the portfolios.

In Section 2 we show that if securities are combined randomly into portfolios, the diversification effect leads to an identical speed of shrinkage of profits from both the systematic and the idiosyncratic components of returns. To examine if the corresponding momentum profits decrease at the rate of $\frac{1}{n}$ and approach zero, we implement momentum strategies using portfolios of securities combined *randomly* from the entire population. Specifically, we generate a uniform random number for each stock present in a ranking period, and sort stocks by the magnitude of the random number.

[13] This is inconsistent with the negative estimates of the autocorrelation and autocovariance presented in Jegadeesh and Titman (1993) and Moskowitz and Grinblatt (1999), partly because these parameters are downward biased especially in their much smaller post-1962 samples.

Base portfolios of size n are then formed by sequentially picking off n stocks at a time; the n stocks with the largest random numbers are grouped in one base portfolio, the next n stocks in another base portfolio, repeating this procedure until all stocks are assigned to portfolios.[14] Then holding the set of uniform random numbers fixed, we form base portfolios of size $n+1$ by sequentially picking off $n+1$ stocks at a time, and so on.

If continuation in the idiosyncratic component of returns is the sole (or predominant) reason for the profitability of momentum strategies, the profits of these strategies will again decline at the rate of $\frac{1}{n}$ [see (15)]. As in the case of the portfolios based on relative strength, this will occur because the firm-specific components of two securities cannot be correlated with each other over time. However, contrary to portfolios based on relative strength, even if cross-sectional variation in expected returns and risks is the source of the profitability of momentum strategies, profits of these random portfolio strategies will also decline at the rate of $\frac{1}{n}$ to zero for very large portfolios. The difference in profit patterns arises from the fact that, unlike the relative strength portfolio strategies, the random portfolio strategies in this experiment fail to maintain the cross-sectional variation in mean returns and risks. By combining securities randomly into portfolios we generate a level of average cross-products within portfolios that is close to the average cross-products for all securities, and the diversification effect forces the systematic component of returns to shrink at the speed of $\frac{1}{n}$. Consequently, the expected profits of the random portfolio strategies should exhibit a dramatically different pattern than the one exhibited both theoretically and empirically by portfolios created from securities ranked based on their past relative strength.

The tests based on the equal-weighted random portfolios are presented in Table 5.[15] Panel A reports average profits for portfolios of different sizes and, for convenience, we also report the profits of the single-security strategy in the first row. To gauge the statistical significance of the various average estimates, we present the heteroskedasticity and autocorrelation consistent t-statistics. Consistent with the prediction of the experimental design, the estimated profits in the overall period decline at the rate of $\frac{1}{n}$. A similar pattern can be witnessed during each sub-period. This is in stark contrast to the profits of momentum strategies that use portfolios of securities based on their past relative strength (see Table 3, Panel A). The comparison of the estimates in Panels A of Tables 3 and 5 confirms that the profitability of momentum strategies reported in the literature is due to the cross-sectional variation in the systematic component of security returns because that is the only difference between the two sets of portfolios used in these different strategies.

Table 5, Panel B reports the average investment involved in the strategies. Note that within each sample period, the average investment estimates decline with n, which is in contrast to the unchanged estimates of investments in Panel B of Table 3. This however is not surprising because,

[14] Again, the presence of base portfolios with unbalanced size has virtually no impact on our results. Requiring exactly n stocks for each base portfolio in the ranking period, or requiring exactly n stocks in each base portfolio in both the ranking and holding periods, does not alter our evidence.

[15] We again do not present the value-weighted results for brevity and also because the theoretical predictions are not as sharp as for the equal-weighted portfolios given the lack of a clear theoretical pattern for the cross-sectional distribution of value weights. The results however are qualitatively similar.

unlike the strategies with portfolios grouped using securities rank ordered based on past performance, in this experiment the securities are combined randomly into portfolios. It can be shown that the expected investment, since it is based on the absolute deviations of individual-security returns from the return of the equal-weighted market portfolio [see (2)], will decline with n in these portfolios. An unintended consequence of this decline is that the scaled momentum profits decline at a slower rate than predicted by the set up of the experiment, which again demonstrates that the scaling process is not innocuous. As mentioned earlier, caution needs to be exercised in using scaled profits to make comparisons across different trading strategies.

Table 6 contains estimated average own- and cross-products of the randomly generated base portfolios to shed light on the dynamics of these momentum strategies, with the corresponding heteroskedasticity and autocorrelation consistent t-statistics. Not surprisingly, Panel A of Table 6 is identical to Panel A of Table 4 because the individual securities that form the basis of both sets of strategies are identical. As before, Panel A shows that the importance of average own-products declines at the rate of $\frac{1}{n}$. This also confirms that the average estimates of the own-products cannot help distinguish between alternate sources of momentum profits. The interesting aspect of Table 6 again lies in the behavior of the un-weighted average cross-products of the portfolios in Panel C, relative to the average cross-products of all securities in the entire universe (see Panel D). Recall that the average profits will decline with n for the random portfolios and, unlike the portfolios containing securities rank ordered based on their past performance, the profits will not remain unaltered. This will occur even for small n because the averages of the un-weighted cross-products *within* the base portfolios will converge to the average cross-products of all securities in the sample. A comparison of the estimates in Panels C and D of Table 6 shows that even for $n = 2$ the average estimate of the un-weighted cross-products within the portfolio (0.867×10^{-2}) is very close to, and statistically indistinguishable from, the average estimate of the cross-products of all securities in the universe (0.810×10^{-2}). Since the average of the un-weighted cross-products within the base portfolios remain roughly the same with an increase in the size of the portfolios, the average profits of the momentum strategies decline at the rate of $\frac{1}{n}$ to zero. Consequently, the lack of cross-sectional variation in expected returns and risks of the portfolios of randomly combined securities eventually leads to a lack of profitability of the strategies for reasonably large n.

4 Summary and Conclusions

In this paper, we develop a new methodology to test whether momentum is indeed an anomaly. Our methodology is unique in that it bypasses the need to specify and estimate a specific model of expected returns. More importantly, it does not require a decomposition of momentum profits, a process that has riddled past studies with model specification and estimation problems. Our methodology and tests are based instead on the appealing logic and intuition of the simple observation that repeated ranking of stocks based on their relative strength should have some relation with the expected returns and risks of the securities used in the strategies. And using portfolios of stocks with similar rank ordering based on their past performance versus randomly chosen stocks should help us distinguish between the rational and irrational sources of momentum profits.

We believe that a secondary and reassuring contribution of our paper is that we present compelling evidence that momentum is not an anomaly. More specifically, there is no evidence that under- (or continued over-) reaction to firm-specific news is the cause for the remarkable and continued success of momentum strategies at intermediate horizons. On the contrary, momentum appears to be a perfectly understandable phenomenon in a world in which different securities have different expected returns and risks. Our results are reassuring because if momentum were to arise solely or predominantly from irrational serial covariance in the firm-specific components of returns, it would not only violate market efficiency in its weakest form but also bring into question the reasonable belief that individuals are risk-averse and therefore demand a (rather large?) risk premium.

Our main goal in this paper is to address the important issue of whether momentum is an anomaly or not. We therefore have not tried to determine the sources of the seasonal patterns in momentum. Specifically, we have not addressed why momentum is not present in the December-January part of the year. We believe that this variation within the year is not the main part of the momentum puzzle and convincing evidence based on tax-loss selling has already been presented by Grinblatt and Moskowitz (2004) to explain the seasonality. Another important issue that is not addressed in this paper is the apparent reversal of the momentum effect in the long run. Addressing this issue is part of our ongoing research agenda of studying the profitability of different strategies implemented over different time horizons using the methodology introduced in this paper.

One final caveat is in order. Our interpretation that momentum is not an anomaly is clearly contextual, because it is conceivable that any cross-sectional patterns in returns could be generated by behavioral biases rather than legitimate differences in expected returns and risks of individual securities. It is however obvious from the development of the literature that momentum is viewed as anomalous because it is believed to be an idiosyncratic phenomenon. Our paper establishes that there is no momentum in the idiosyncratic components of security returns.

A reasonable criticism of our paper however remains that we do not present a specific model of expected returns that could generate the varying time-series behavior of momentum profits. While a fair criticism, we believe that the determination of an asset-pricing model that can explain the richness and variability of momentum profits, as well as other patterns in the data, remains probably the most significant challenge for our profession. And even if we were to develop such a model, the issue of estimation using small samples of medium-horizon returns would continue to pose significant challenges to making reliable inferences. It is in this context that we believe that the methodology, as well as the evidence, presented in this paper should help further our understanding of both the cross-sectional and time-series behavior of asset prices.

5 Appendix: Profits to Portfolio-based Momentum Strategies

Consider a market with N stocks and a momentum strategy on the basis of portfolios, each of which consists of exactly n individual stocks. Without loss of generality, assume that N can be evenly divided by n so that the momentum strategy concerns N/n base portfolios. Focus on the equal-weighting scheme for the base portfolio returns as well as the momentum portfolio return, so that

$$r_{p,t} = \frac{1}{n}\sum_{i=1}^{n} r_{i,t}; w_{p,t} = \frac{1}{N/n}\left(r_{p,t-1} - \frac{1}{N}\sum_{i=1}^{N} r_{i,t-1}\right). \tag{A1}$$

It can be shown that under the single factor return structure (4)-(8), the portfolio-based momentum strategy generates the following expected return

$$\begin{aligned}
E(\pi_{pt}\,|_{n\geq 2}) \quad = \quad & \frac{n}{N}\sum_{p=1}^{N/n}\left\{\left[\frac{1}{n}\sum_{i=1}^{n}\mu_{p,i}\right]\left[\frac{1}{n}\sum_{i=1}^{n}\mu_{p,i}\right]\right\} \\
& -\frac{n^2}{N^2}\sum_{p=1}^{N/n}\left[\frac{1}{n}\sum_{i=1}^{n}\mu_{p,i}\right]\sum_{k=1}^{N/n}\left[\frac{1}{n}\sum_{i=1}^{n}\mu_{k,i}\right] \\
& +\frac{n}{N}\sum_{p=1}^{N/n}\left\{\left[\frac{1}{n}\sum_{i=1}^{n}\beta_{p,i}\right]\left[\frac{1}{n}\sum_{i=1}^{n}\beta_{p,i}\right]\right\}Cov(f_t, f_{t-1}) \\
& -\frac{n^2}{N^2}\sum_{p=1}^{N/n}\left[\frac{1}{n}\sum_{i=1}^{n}\beta_{p,i}\right]\sum_{k=1}^{N/n}\left[\frac{1}{n}\sum_{i=1}^{n}\beta_{k,i}\right]Cov(f_t, f_{t-1}) \\
& +\left[\left(\frac{n}{N} - \frac{n^2}{N^2}\right)\frac{1}{n^2}\right]\sum_{i=1}^{N} Cov(\varepsilon_{it}, \varepsilon_{i,t-1}),
\end{aligned} \tag{A2}$$

where stock i in base portfolio p is denoted by the subscript (p, i). Note that the equation above degenerates to equation (10) when setting $n = 1$.

Breaking the products of sum into sum of self products and cross products, we get

$$\begin{aligned}
E(\pi_{pt}\,|_{n\geq 2}) \quad = \quad & \frac{1}{Nn}\left\{N\overline{\mu_i^2}^{ALL} + \frac{N}{n}n(n-1)\overline{\mu_i\mu_j}^{PORT}\right\} \\
& -\frac{1}{N^2}\left\{N\overline{\mu_i^2}^{ALL} + N(N-1)\overline{\mu_i\mu_j}^{ALL}\right\} \\
& +\frac{1}{Nn}\left\{N\overline{\beta_i^2}^{ALL} + \frac{N}{n}n(n-1)\overline{\beta_i\beta_j}^{PORT}\right\}Cov(f_t, f_{t-1}) \\
& -\frac{1}{N^2}\left\{N\overline{\beta_i^2}^{ALL} + N(N-1)\overline{\beta_i\beta_j}^{ALL}\right\}Cov(f_t, f_{t-1}) \\
& +\left(\frac{1}{n} - \frac{1}{N}\right)\overline{Cov(\varepsilon_{it}, \varepsilon_{i,t-1})}^{ALL},
\end{aligned} \tag{A3}$$

where a few shorthands are defined as

$$\begin{aligned}
\overline{\mu_i^2}^{ALL} &\equiv \frac{1}{N}\sum_{i=1}^{N}\mu_i^2,\ \overline{\beta_i^2}^{ALL} \equiv \frac{1}{N}\sum_{i=1}^{N}\beta_i^2,\ \overline{Cov(\varepsilon_{it},\varepsilon_{i,t-1})}^{ALL} \equiv \frac{1}{N}\sum_{i=1}^{N}Cov(\varepsilon_{it},\varepsilon_{i,t-1}),\\
\overline{\mu_i\mu_j}^{ALL} &\equiv \frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{j\neq i}^{N}\mu_i\mu_j,\ \overline{\beta_i\beta_j}^{ALL} \equiv \frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{j\neq i}^{N}\beta_i\beta_j,\\
\overline{\mu_i\mu_j}^{PORT} &\equiv \frac{1}{N(n-1)}\sum_{p=1}^{N/n}\sum_{i=1}^{n}\sum_{j\neq i}^{n}\mu_{p,i}\mu_{p,j},\ \text{and}\ \overline{\beta_i\beta_j}^{PORT} \equiv \frac{1}{N(n-1)}\sum_{p=1}^{N/n}\sum_{i=1}^{n}\sum_{j\neq i}^{n}\beta_{p,i}\beta_{p,j}.
\end{aligned}$$

The equation above documents separately the number of own products and self products. For instance, there are $(N/n)n(n-1)$ cross products within N/n base portfolios, and $N(N-1)$ cross products among all stocks. Collecting terms in the above equation, we have

$$\begin{aligned}
E(\pi_{pt}\,|_{n\geq 2}) &= \left[\left(\frac{1}{n}-\frac{1}{N}\right)\overline{\mu_i^2}^{ALL} + \frac{n-1}{n}\overline{\mu_i\mu_j}^{PORT} - \frac{N-1}{N}\overline{\mu_i\mu_j}^{ALL}\right]\\
&+ \left[\left(\frac{1}{n}-\frac{1}{N}\right)\overline{\beta_i^2}^{ALL} + \frac{n-1}{n}\overline{\beta_i\beta_j}^{PORT} - \frac{N-1}{N}\overline{\beta_i\beta_j}^{ALL}\right]Cov(f_t,f_{t-1}) \quad \text{(A4)}\\
&+ \left(\frac{1}{n}-\frac{1}{N}\right)\overline{Cov(\varepsilon_{it},\varepsilon_{i,t-1})}^{ALL}.
\end{aligned}$$

For very large N, we have $1/N \approx 0$ and $1/N^2 \approx 0$. Therefore, we obtain equations (13) and (14).

References

[1] Ahn, Dong-Hyun, Jennifer Conrad, and Robert F. Dittmar, 2003, Risk Adjustment and Trading Strategies, *Review of Financial Studies* 16, 459–485.

[2] Bansal, Ravi, Robert F. Dittmar, and Christian Lundblad, 2005, Consumption, Dividends, and the Cross-Section of Equity Returns, *Journal of Finance* 60, 1639–1672.

[3] Asness, Clifford, R. Burt Porter, and Ross Stevens, 2000, Predicting Stock Returns Using Industry-Relative Firm Characteristics, *Iowa State University Working Paper.*

[4] Barberis, Nicholas, Andrei Shleifer, and Robert Vishny, 1998, A Model of Investor Sentiment, *Journal of Financial Economics* 49, 307-343.

[5] Barberis, Nicholas, and Richard Thaler, 2003, A Survey of Behavioral Finance, in George M. Constantinides, Milton Harris, and René Stulz, eds.: *Handbook of the Economics of Finance* (Elsevier, Amsterdam).

[6] Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal Investment, Growth Options, and Security Returns, *Journal of Finance* 54, 1553-1607.

[7] Black, Fischer, 1993, Beta and Return, *Journal of Portfolio Management* 20, 8-18.

[8] Brown, Stephen J., and Jerold B. Warner, 1980, Measuring Security Price Performance, *Journal of Financial Economics* 8, 205-258.

[9] Brown, Stephen J., and Jerold B. Warner, 1985, Using Daily Stock Returns: The Case of Event Studies, *Journal of Financial Economics* 14, 3-31.

[10] Chan, Louis K. C., Narasimhan Jegadeesh, and Josef Lakonishok, 1996, Momentum Strategies, *Journal of Finance* 51, 1681-1713.

[11] Chen, Joseph, and Harrison Hong, 2002, Discussion Of "Momentum and Autocorrelation in Stock Returns", *Review of Financial Studies* 15, 565-573.

[12] Chordia, Tarun, and Lakshmanan Shivakumar, 2002, Momentum, Business Cycle, and Time-Varying Expected Returns, *Journal of Finance* 57, 985-1019.

[13] Cochrane, John H., 1996, A Cross-Sectional Test of an Investment-Based Asset Pricing Model, *Journal of Political Economy* 104, 572-621.

[14] Conrad, Jennifer, and Gautam Kaul, 1998, An Anatomy of Trading Strategies, *Review of Financial Studies* 11, 489-519.

[15] Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 1998, Investor Psychology and Security Market under- and Overreactions, *Journal of Finance* 53, 1839-1885.

[16] Davidson, Russell, and James G. MacKinnon, 1993. *Estimation and Inference in Econometrics* (Oxford University Press, New York).

[17] de Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann, 1990, Positive Feedback Investment Strategies and Destabilizing Rational Speculation, *Journal of Finance* 45, 379-395.

[18] Dixon, Wilfrid J., 1944, Further Contributions to the Problem of Serial Correlation, *Annals of Mathematical Statistics* 15, 119-144.

[19] Fama, Eugene F., 1970, Efficient Capital Markets: A Review of Theory and Empirical Work, *Journal of Finance* 25, 383-417.

[20] Fama, Eugene F., 1998, Market Efficiency, Long-Term Returns, and Behavioral Finance, *Journal of Financial Economics* 49, 283-306.

[21] Fama, Eugene F., and Kenneth R. French, 1992, The Cross-Section of Expected Stock Returns, *Journal of Finance* 47, 427-465.

[22] Fama, Eugene F., and Kenneth R. French, 1993, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics* 33, 3-56.

[23] Fama, Eugene F., and Kenneth R. French, 1996, Multifactor Explanations of Asset Pricing Anomalies, *Journal of Finance* 51, 55-84.

[24] Griffin, John M., Xiuqing Ji, and J. Spencer Martin, 2003, Momentum Investing and Business Cycle Risk: Evidence from Pole to Pole, *Journal of Finance* 58, 2515-2547.

[25] Grinblatt, Mark, and Bing Han, 2002, The Disposition Effect and Momentum, *NBER Working Paper*.

[26] Grinblatt, Mark, and Tobias J. Moskowitz, 2004, Predicting Stock Price Movements from Past Returns: The Role of Consistency and Tax-Loss Selling, *Journal of Financial Economics* 71, 541-579.

[27] Grundy, Bruce D., and J. Spencer Martin, 2001, Understanding the Nature of the Risks and the Source of the Rewards to Momentum Investing, *Review of Financial Studies* 14, 29-78.

[28] Hong, Harrison, Terence Lim, and Jeremy C. Stein, 2000, Bad News Travels Slowly: Size, Analyst Coverage, and the Profitability of Momentum Strategies, *Journal of Finance* 55, 265-295.

[29] Hong, Harrison, and Jeremy C. Stein, 1999, A Unified Theory of Underreaction, Momentum Trading, and Overreaction in Asset Markets, *Journal of Finance* 54, 2143-2184.

[30] Hou, Kewei, and Tobias J. Moskowitz, 2005, Market Frictions, Price Delay, and the Cross-Section of Expected Returns, *Review of Financial Studies* 18, 981-1020.

[31] Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency, *Journal of Finance* 48, 65-91.

[32] Jegadeesh, Narasimhan, and Sheridan Titman, 2001, Profitability of Momentum Strategies: An Evaluation of Alternative Explanations, *Journal of Finance* 56, 699-720.

[33] Jegadeesh, Narasimhan, and Sheridan Titman, 2002, Cross-Sectional and Time-Series Determinants of Momentum Returns, *Review of Financial Studies* 15, 143-157.

[34] Jegadeesh, Narasimhan, and Sheridan Titman, 2005, Momentum, in Richard M. Thaler, ed.: *Advances in Behavioral Finance* Volume II (Princeton University Press, Princeton).

[35] Johnson, Timothy C., 2002, Rational Momentum Effects, *Journal of Finance* 57, 585-608.

[36] Korajczyk, Robert A., and Ronnie Sadka, 2004, Are Momentum Profits Robust to Trading Costs?, *Journal of Finance* 59, 1039-1082.

[37] Lakonishok, Josef, Andrei Shleifer, and Robert W. Vishny, 1994, Contrarian Investment, Extrapolation, and Risk, *Journal of Finance* 49, 1541-1578.

[38] Lee, Charles M. C., and Bhaskaran Swaminathan, 2000, Price Momentum and Trading Volume, *Journal of Finance* 55, 2017-2069.

[39] Lehmann, Bruce N., 1990, Fads, Martingales, and Market Efficiency, *Quarterly Journal of Economics* 105, 1-28.

[40] Lewellen, Jonathan, 2002, Momentum and Autocorrelation in Stock Returns, *Review of Financial Studies* 15, 533-563.

[41] Liu, Laura Xiaolei, and Lu Zhang, 2007, Momentum Profits, Factor Pricing, and Macroeconomic Risk, *University of Michigan Working Paper*.

[42] Lo, Andrew W., and A. Craig MacKinlay, 1990, When Are Contrarian Profits Due to Stock Market Overreaction?, *Review of Financial Studies* 3, 175-205.

[43] Mehra, Rajnish, and Edward C. Prescott, 1985, The Equity Premium: A Puzzle, *Journal of Monetary Economics* 15, 145-161.

[44] Merton, Robert C., 1980, On Estimating the Expected Return on the Market : An Exploratory Investigation, *Journal of Financial Economics* 8, 323-361.

[45] Moskowitz, Tobias J., and Mark Grinblatt, 1999, Do Industries Explain Momentum?, *Journal of Finance* 54, 1249-1290.

[46] Newey, Whitney K., and Kenneth D. West, 1987, A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* 55, 703-708.

[47] Rouwenhorst, K. Geert, 1998, International Momentum Strategies, *Journal of Finance* 53, 267-284.

[48] Sadka, Ronnie, 2006, Momentum and Post-Earnings-Announcement Drift Anomalies: The Role of Liquidity Risk, *Journal of Financial Economics* 80, 309-349.

[49] Wu, Xueping, 2002, A Conditional Multifactor Analysis of Return Momentum, *Journal of Banking & Finance* 26, 1675-1696.

[50] Wu, Ding, 2007, An Adverse Selection Explanation of Momentum: Theory and Evidence, *Massachusetts Institute of Technology Working Paper*.

Table 1
Momentum Profits Using Single-Security Strategies

This table reports the results from implementing momentum strategies using all stocks on the NYSE and AMEX between 1926 and 2006. Monthly returns of individual stocks are first compounded over a six-month ranking period. These returns are then compared to a benchmark return (either equal-weighted or value-weighted market return) to separate winners from losers. The proceeds from selling losers are used to buy winners, with extreme winners/losers receiving the highest weights [see (1) for the equal-weighted strategy and (17) for the value-weighted strategy]. The resulting portfolio is held for the subsequent six-month holding period. The momentum profit is computed based on the compounded returns for individual stocks during the holding period. We maintain a one-month gap between the ranking and holding periods, in order to mitigate market microstructure issues. This procedure is repeated in a rolling fashion for the entire sample period, and the time stamp of momentum profits ranges from July, 1926 to June, 2006. In the following table, the four-digit numbers in the headers refer to the year and the month of the stamped time, i.e., the month in the gap. For each 13-month study period, we keep track of the momentum profit (π), the dollar investments on the long position (lp), and the momentum profit scaled by the long position (π/lp). The time series averages of these three measures are reported below, along with heteroskedasticity and autocorrelation consistent t-statistics in parentheses. Note that profits are multiplied by 100.

	Full Sample	Sub Sample Period			
	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606
		Panel (A) Equal-weighted Strategy			
π	0.417	0.371	0.284	0.463	0.549
	(6.506)	(2.018)	(6.003)	(4.665)	(3.963)
lp	0.094	0.110	0.066	0.099	0.101
	(38.860)	(14.194)	(34.354)	(37.028)	(45.145)
π/lp	4.377	2.818	4.556	4.656	5.479
	(7.706)	(1.848)	(7.835)	(4.776)	(4.492)
		Panel (B) Value-weighted Strategy			
π	0.192	0.277	0.170	0.163	0.157
	(4.837)	(2.768)	(4.987)	(2.300)	(1.672)
lp	0.065	0.071	0.050	0.067	0.071
	(43.131)	(15.941)	(38.144)	(36.938)	(28.346)
π/lp	3.065	3.298	3.630	2.745	2.588
	(6.257)	(2.664)	(5.587)	(2.914)	(2.600)

Table 2
Momentum Profits to Equal-weighted Strategies Using Ranking-Return-Portfolios

This table reports the results from implementing portfolio-based momentum strategies using all stocks on the NYSE/AMEX between 1926 and 2006. Monthly returns of individual stocks are first compounded over a six-month ranking period. These returns are then compared to the equal-weighted market return to separate winners from losers. Stocks are then sorted by their strength within each category, with the strongest winning/losing stocks being ranked first. Base portfolios of size n are then formed by sequentially picking off n stocks at a time from each category. Specifically, the strongest n winning (losing) stocks are grouped in one winner (loser) portfolio; the next n winning (losing) stocks are grouped in the next winner (loser) portfolio; repeating this procedure until all winning (losing) stocks are part of some winner (loser) portfolio. We compute equal-weighted returns for the base portfolios in the ranking period and compare them to the equal-weighted market return to separate winner portfolios from losers. The proceeds from selling loser portfolios are used to buy winner portfolios, with extreme winners/losers receiving the highest weights [see (11) for the weighting function]. The resulting momentum portfolio is held for the subsequent six-month holding period. The momentum profit is calculated from the equal-weighted base portfolio returns for the holding period. We maintain a one-month gap between the ranking and holding periods, in order to mitigate any market microstructure issues. This procedure is repeated in a rolling fashion for the entire sample period, and the time stamp of momentum profits range from July, 1926 to June, 2006. We allow for ten different sizes for base portfolios, that is, n ranges from 1, 2, 3, 4, 5, 10, 20, 30, 40 to 50. The four-digit numbers in the headers of the table refer to the year and the month of the stamped time, i.e., the month in the gap. For each 13-month study period, we keep track of the momentum profit (π), the dollar investments on the long position (lp), and the momentum profit scaled by the long position, (π/lp). The time-series averages of these three measures are reported below, along with heteroskedasticity and autocorrelation consistent t-statistics. Note that profits are multiplied by 100.

	Full Sample	Sub Sample Period				Full Sample	Sub Sample Period			
	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606
n	Panel (A) Momentum Profit $\pi(n)$					Newey-West t-statistic				
1	0.417	0.371	0.284	0.463	0.549	6.506	2.018	6.003	4.665	3.963
2	0.436	0.344	0.288	0.509	0.601	6.730	1.873	6.059	4.967	4.298
3	0.444	0.355	0.290	0.515	0.618	6.701	1.909	6.063	5.014	4.176
4	0.441	0.350	0.290	0.515	0.608	6.776	1.893	6.067	5.029	4.335
5	0.439	0.348	0.292	0.517	0.599	6.810	1.910	6.084	5.035	4.296
10	0.431	0.336	0.293	0.515	0.581	6.888	1.871	6.085	5.028	4.492
20	0.422	0.316	0.291	0.515	0.565	6.977	1.823	6.068	5.014	4.670
30	0.418	0.306	0.287	0.515	0.563	7.025	1.799	6.054	5.022	4.777
40	0.415	0.292	0.287	0.516	0.565	7.110	1.760	6.111	5.041	4.902
50	0.413	0.289	0.285	0.516	0.562	7.049	1.727	6.102	5.032	4.900

	Full Sample	Sub Sample Period				Full Sample	Sub Sample Period			
	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606
n	Panel (B) Long Position $lp(n)$					Newey-West t-statistic				
1	0.094	0.110	0.066	0.099	0.101	38.860	14.194	34.354	37.028	45.145
2	0.094	0.110	0.066	0.099	0.101	38.860	14.194	34.354	37.028	45.145
3	0.094	0.110	0.066	0.099	0.101	38.860	14.194	34.354	37.028	45.145
4	0.094	0.110	0.066	0.099	0.101	38.860	14.194	34.354	37.028	45.145
5	0.094	0.110	0.066	0.099	0.101	38.860	14.194	34.354	37.028	45.145
10	0.094	0.110	0.066	0.099	0.101	38.860	14.194	34.354	37.028	45.145
20	0.094	0.110	0.066	0.099	0.101	38.860	14.194	34.354	37.028	45.145
30	0.094	0.110	0.066	0.099	0.101	38.860	14.194	34.354	37.028	45.145
40	0.094	0.110	0.066	0.099	0.101	38.860	14.194	34.354	37.028	45.145
50	0.094	0.110	0.066	0.099	0.101	38.860	14.194	34.354	37.028	45.145
n	Panel (C) Scaled Momentum Profit $\pi(n)/lp(n)$					Newey-West t-statistic				
1	4.377	2.818	4.556	4.656	5.479	7.706	1.848	7.835	4.776	4.492
2	4.593	2.606	4.625	5.141	6.001	7.910	1.677	7.928	5.146	4.840
3	4.673	2.680	4.649	5.207	6.155	7.904	1.723	7.916	5.191	4.702
4	4.649	2.649	4.647	5.210	6.091	7.963	1.693	7.890	5.221	4.880
5	4.645	2.690	4.669	5.223	5.996	7.987	1.727	7.910	5.238	4.826
10	4.591	2.614	4.678	5.220	5.852	8.072	1.697	7.884	5.231	5.027
20	4.513	2.481	4.658	5.204	5.708	8.128	1.647	7.858	5.212	5.181
30	4.481	2.452	4.597	5.213	5.662	8.162	1.647	7.861	5.213	5.251
40	4.470	2.363	4.602	5.221	5.694	8.217	1.604	7.939	5.239	5.368
50	4.435	2.285	4.569	5.224	5.663	8.159	1.553	7.913	5.232	5.353

Table 3
Momentum Profits to Value-weighted Strategies Using Ranking-Return-Portfolios

This table reports the results from implementing portfolio-based momentum strategies using all stocks on the NYSE/AMEX between 1926 and 2006. Monthly returns of individual stocks are first compounded over a six-month ranking period. These returns are then compared to the value-weighted market return to separate winners from losers. Stocks are then sorted by their strength within each category, with the strongest winning/losing stocks being ranked first. Base portfolios of size n are then formed by sequentially picking off n stocks at a time from each category. Specifically, the strongest n winning (losing) stocks are grouped in one winner (loser) portfolio; the next n winning (losing) stocks are grouped in the next winner (loser) portfolio; repeating this procedure until all winning (losing) stocks are part of some winner (loser) portfolio. We compute value-weighted returns for the base portfolios in the ranking period and compare them to the value-weighted market return to separate winner portfolios from losers. The proceeds from selling loser portfolios are used to buy winner portfolios, with extreme winners/losers receiving the highest weights [see (17) for the weighting function substituting the value-weighted base portfolio returns for the individual stock returns]. The resulting momentum portfolio is held for the subsequent six-month holding period. The momentum profit is calculated from the value-weighted base portfolio returns for the holding period. We maintain a one-month gap between the ranking and holding periods, in order to mitigate any market microstructure issues. This procedure is repeated in a rolling fashion for the entire sample period, and the time stamp of momentum profits range from July, 1926 to June, 2006. We allow for ten different sizes for base portfolios, that is, n ranges from 1, 2, 3, 4, 5, 10, 20, 30, 40 to 50. The four-digit numbers in the headers of the table refer to the year and the month of the stamped time, i.e., the month in the gap. For each 13-month study period, we keep track of the momentum profit (π), the dollar investments on the long position (lp), and the momentum profit scaled by the long position, (π/lp). The time-series averages of these three measures are reported below, along with heteroskedasticity and autocorrelation consistent t-statistics. Note that profits are multiplied by 100.

	Full Sample	Sub Sample Period				Full Sample	Sub Sample Period			
	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606
n	Panel (A) Momentum Profit $\pi(n)$					Newey-West t-statistic				
1	0.192	0.277	0.170	0.163	0.157	4.837	2.768	4.987	2.300	1.672
2	0.199	0.276	0.170	0.181	0.167	4.972	2.765	4.981	2.502	1.753
3	0.201	0.280	0.171	0.179	0.174	5.076	2.819	5.006	2.474	1.861
4	0.200	0.278	0.171	0.178	0.172	5.031	2.776	4.988	2.459	1.845
5	0.200	0.279	0.171	0.178	0.170	5.037	2.792	5.002	2.445	1.836
10	0.199	0.274	0.172	0.176	0.174	5.026	2.753	5.038	2.419	1.868
20	0.201	0.276	0.173	0.176	0.177	5.121	2.830	5.048	2.425	1.923
30	0.200	0.277	0.170	0.176	0.177	5.130	2.851	4.982	2.411	1.941
40	0.200	0.270	0.171	0.178	0.181	5.159	2.807	5.046	2.449	1.986
50	0.199	0.269	0.170	0.178	0.180	5.158	2.827	5.047	2.435	1.977

	Full Sample	Sub Sample Period				Full Sample	Sub Sample Period			
	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606
n	Panel (B) Long Position $lp(n)$					Newey-West t-statistic				
1	0.065	0.071	0.050	0.067	0.071	43.131	15.941	38.144	36.938	28.346
2	0.065	0.071	0.050	0.067	0.071	43.131	15.941	38.144	36.938	28.346
3	0.065	0.071	0.050	0.067	0.071	43.131	15.941	38.144	36.938	28.346
4	0.065	0.071	0.050	0.067	0.071	43.131	15.941	38.144	36.938	28.346
5	0.065	0.071	0.050	0.067	0.071	43.131	15.941	38.144	36.938	28.346
10	0.065	0.071	0.050	0.067	0.071	43.131	15.941	38.144	36.938	28.346
20	0.065	0.071	0.050	0.067	0.071	43.131	15.941	38.144	36.938	28.346
30	0.065	0.071	0.050	0.067	0.071	43.131	15.941	38.144	36.938	28.346
40	0.065	0.071	0.050	0.067	0.071	43.131	15.941	38.144	36.938	28.346
50	0.065	0.071	0.050	0.067	0.071	43.131	15.941	38.144	36.938	28.346
n	Panel (C) Scaled Momentum Profit $\pi(n)/lp(n)$					Newey-West t-statistic				
1	3.065	3.298	3.630	2.745	2.588	6.257	2.664	5.587	2.914	2.600
2	3.172	3.285	3.643	3.032	2.727	6.456	2.673	5.584	3.171	2.714
3	3.209	3.346	3.658	3.010	2.822	6.540	2.728	5.607	3.143	2.818
4	3.191	3.316	3.658	2.992	2.799	6.484	2.679	5.586	3.131	2.800
5	3.193	3.329	3.660	2.998	2.786	6.491	2.695	5.607	3.111	2.801
10	3.186	3.280	3.682	2.959	2.824	6.475	2.655	5.643	3.080	2.827
20	3.206	3.307	3.695	2.961	2.863	6.580	2.739	5.660	3.080	2.879
30	3.183	3.302	3.643	2.953	2.832	6.566	2.763	5.605	3.063	2.866
40	3.181	3.183	3.655	2.994	2.893	6.596	2.687	5.669	3.107	2.933
50	3.158	3.146	3.639	2.981	2.868	6.579	2.686	5.675	3.088	2.909

Table 4
Profit Decomposition of Equal-weighted Strategies Using Ranking-Return-Portfolios

This table reports the results from implementing portfolio-based momentum strategies using all stocks on the NYSE/AMEX between 1926 and 2006. Monthly returns of individual stocks are first compounded over a six-month ranking period. These returns are then compared to the equal-weighted market return to separate winners from losers. Stocks are then sorted by their strength within each category, with the strongest winning/losing stocks being ranked first. Base portfolios of size n are then formed by sequentially picking off n stocks at a time from each category. Specifically, the strongest n winning (losing) stocks are grouped in one winner (loser) portfolio; the next n winning (losing) stocks are grouped in the next winner (loser) portfolio; repeating this procedure until all winning (losing) stocks are part of some winner (loser) portfolio. We compute equal-weighted returns for the base portfolios in the ranking period and compare them to the equal-weighted market return to separate winner portfolios from losers. The proceeds from selling loser portfolios are used to buy winner portfolios, with extreme winners/losers receiving the highest weights [see (11) for the weighting function]. The resulting momentum portfolio is held for the subsequent six-month holding period. The momentum profit is calculated from the equal-weighted base portfolio returns for the holding period. We maintain a one-month gap between the ranking and holding periods, in order to mitigate any market microstructure issues. This procedure is repeated in a rolling fashion for the entire sample period, and the time stamp of momentum profits range from July, 1926 to June, 2006. We allow for ten different sizes for base portfolios, that is, n ranges from 1, 2, 3, 4, 5, 10, 20, 30, 40 to 50. The four-digit numbers in the headers of the table refer to the year and the month of the stamped time, i.e., the month in the gap. For each 13-month study period, we keep track of the weighted sum of own-products of base portfolio returns, the weighted sum of cross-products of base portfolio returns, the un-weighted sum of cross-products of base portfolio returns, and the own-products of equal-weighted market returns. Note that all the own- and cross-products are defined as the products of returns in the ranking and holding periods, multiplied by 100. The time series averages of these four measures are reported below, along with heteroskedasticity and autocorrelation consistent t-statistics.

	Full Sample	Sub Sample Period				Full Sample	Sub Sample Period			
	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606
n	Panel (A) Weighted Own-products of Base Portfolios					Newey-West t-statistic				
1	1.239	2.547	0.769	0.957	0.684	3.546	2.011	3.404	2.187	2.557
2	0.620	1.274	0.384	0.478	0.342	3.546	2.011	3.404	2.187	2.557
3	0.413	0.849	0.256	0.319	0.228	3.546	2.011	3.404	2.187	2.557
4	0.310	0.637	0.192	0.239	0.171	3.546	2.011	3.404	2.187	2.557
5	0.248	0.509	0.154	0.191	0.137	3.546	2.011	3.404	2.187	2.557
10	0.124	0.255	0.077	0.096	0.068	3.546	2.011	3.404	2.187	2.557
20	0.062	0.127	0.038	0.048	0.034	3.546	2.011	3.404	2.187	2.557
30	0.041	0.085	0.026	0.032	0.023	3.546	2.011	3.404	2.187	2.557
40	0.031	0.064	0.019	0.024	0.017	3.546	2.011	3.405	2.187	2.557
50	0.025	0.051	0.015	0.019	0.014	3.546	2.011	3.404	2.187	2.557

	Full Sample	Sub Sample Period				Full Sample	Sub Sample Period			
	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606
n	Panel (B) Weighted Cross-products of Base Portfolios					Newey-West t-statistic				
1	0.000	0.000	0.000	0.000	0.000	-	-	-	-	-
2	0.653	1.282	0.392	0.542	0.396	3.666	1.992	3.410	2.365	2.830
3	0.868	1.714	0.522	0.707	0.527	3.654	1.997	3.417	2.341	2.786
4	0.967	1.924	0.586	0.786	0.574	3.629	1.993	3.417	2.325	2.770
5	1.026	2.046	0.626	0.836	0.597	3.626	1.998	3.423	2.320	2.713
10	1.143	2.288	0.704	0.931	0.647	3.608	1.996	3.427	2.310	2.695
20	1.194	2.392	0.741	0.979	0.665	3.603	1.995	3.416	2.300	2.672
30	1.210	2.423	0.749	0.995	0.674	3.605	1.996	3.402	2.301	2.669
40	1.218	2.429	0.756	1.003	0.682	3.614	1.995	3.407	2.302	2.691
50	1.222	2.440	0.758	1.009	0.683	3.611	1.995	3.397	2.302	2.683
n	Panel (C) Un-weighted Cross-products of Base Portfolios					Newey-West t-statistic				
1	0.000	0.000	0.000	0.000	0.000	-	-	-	-	-
2	1.306	2.563	0.784	1.085	0.791	3.666	1.992	3.410	2.365	2.830
3	1.302	2.572	0.783	1.061	0.791	3.654	1.997	3.417	2.341	2.786
4	1.290	2.565	0.781	1.048	0.765	3.629	1.993	3.417	2.325	2.770
5	1.283	2.557	0.783	1.045	0.746	3.626	1.998	3.423	2.320	2.713
10	1.270	2.543	0.782	1.035	0.719	3.608	1.996	3.427	2.310	2.695
20	1.257	2.518	0.780	1.030	0.700	3.603	1.995	3.416	2.300	2.672
30	1.252	2.507	0.775	1.030	0.697	3.605	1.996	3.402	2.301	2.669
40	1.249	2.491	0.775	1.029	0.700	3.614	1.995	3.407	2.302	2.691
50	1.247	2.490	0.773	1.030	0.697	3.611	1.995	3.397	2.302	2.683
n	Panel (D) Own-products of Market Returns					Newey-West t-statistic				
All	0.810	2.171	0.481	0.478	0.110	2.604	1.910	2.425	1.264	0.597

Table 5
Momentum Profits to Equal-weighted Strategies Using Random-Portfolios

This table reports the results from implementing portfolio-based momentum strategies using all stocks on the NYSE/AMEX between 1926 and 2006. Monthly returns of individual stocks are first compounded over a six-month ranking period. We generate a uniform random number for each stock present in the ranking period, and sort stocks by the magnitude of the random number. Base portfolios of size n are then formed by sequentially picking off n stocks at a time. Specifically, n stocks with the largest random numbers are grouped in one base portfolio; the next n stocks are grouped in the next base portfolio; repeating this procedure until all stocks are assigned to portfolios. We compute equal-weighted returns for the base portfolios in the ranking period and compare them to the equal-weighted market return so as to separate winners from losers. The proceeds from selling loser portfolios are used to buy winner portfolios, with extreme winners/losers receiving the highest weights [see (11) for the weighting function]. The resulting momentum portfolio is held for the subsequent six-month holding period. The momentum profit is calculated from the equal-weighted base portfolio returns for the holding period. We maintain a one-month gap between the ranking and holding periods, in order to mitigate any market microstructure issues. This procedure is repeated in a rolling fashion for the entire sample period, and the time stamp of momentum profits range from July, 1926 to June, 2006. We allow for ten different sizes for base portfolios, that is, n ranges from 1, 2, 3, 4, 5, 10, 20, 30, 40 to 50. The four-digit numbers in the headers of the table refer to the year and the month of the stamped time, i.e., the month in the gap. For each 13-month study period, we keep track of the momentum profit (π), the dollar investments on the long position (lp), and the momentum profit scaled by the long position, (π/lp). The time-series averages of these three measures are reported below, along with heteroskedasticity and autocorrelation consistent t-statistics. Note that profits are multiplied by 100.

	Full Sample	Sub Sample Period				Full Sample	Sub Sample Period			
	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606
n	Panel (A) Momentum Profit $\pi(n)$					Newey-West t-statistic				
1	0.417	0.371	0.284	0.463	0.549	6.506	2.018	6.003	4.665	3.963
2	0.222	0.191	0.149	0.241	0.306	6.414	1.966	6.317	4.788	3.839
3	0.145	0.131	0.101	0.160	0.190	6.514	2.046	6.467	4.826	3.808
4	0.112	0.107	0.075	0.118	0.148	6.309	2.012	6.253	4.910	3.892
5	0.076	0.050	0.059	0.084	0.113	5.484	1.196	6.229	4.308	3.843
10	0.037	0.009	0.029	0.046	0.063	4.878	0.412	5.817	4.611	4.079
20	0.017	-0.001	0.017	0.021	0.032	3.548	-0.031	5.834	4.129	4.500
30	0.016	0.017	0.011	0.015	0.022	6.616	2.267	5.487	4.156	4.492
40	0.011	0.007	0.009	0.010	0.017	5.607	1.201	5.664	4.082	4.533
50	0.008	0.003	0.008	0.008	0.012	4.135	0.532	5.342	3.651	3.880

	Full Sample	Sub Sample Period				Full Sample	Sub Sample Period			
	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606
n	Panel (B) Long Position $lp(n)$					Newey-West t-statistic				
1	0.094	0.110	0.066	0.099	0.101	38.860	14.194	34.354	37.028	45.145
2	0.070	0.081	0.049	0.073	0.077	37.586	13.707	32.065	35.731	43.127
3	0.058	0.068	0.040	0.061	0.065	37.569	13.881	31.113	35.270	41.656
4	0.051	0.060	0.035	0.053	0.057	37.635	14.023	29.932	34.951	41.059
5	0.046	0.054	0.032	0.048	0.052	37.224	13.865	29.699	34.490	40.041
10	0.034	0.039	0.023	0.034	0.038	36.277	13.517	27.890	34.972	38.297
20	0.024	0.028	0.017	0.024	0.027	36.412	13.584	26.953	34.681	36.787
30	0.020	0.023	0.014	0.020	0.022	36.805	13.889	27.074	33.748	35.612
40	0.017	0.020	0.012	0.018	0.020	37.536	14.501	26.204	32.499	34.530
50	0.015	0.018	0.011	0.016	0.018	37.597	14.476	26.260	32.707	33.674
n	Panel (C) Scaled Momentum Profit $\pi(n)/lp(n)$					Newey-West t-statistic				
1	4.377	2.818	4.556	4.656	5.479	7.706	1.848	7.835	4.776	4.492
2	3.075	1.788	3.240	3.293	3.980	7.799	1.782	8.292	4.929	4.349
3	2.478	1.633	2.660	2.669	2.950	7.836	1.942	8.380	4.861	4.285
4	2.115	1.339	2.288	2.255	2.577	7.776	1.823	8.372	4.930	4.389
5	1.687	0.773	1.958	1.820	2.198	6.798	1.108	7.974	4.397	4.414
10	1.194	0.361	1.407	1.338	1.671	6.523	0.711	7.363	4.416	4.572
20	0.824	0.127	1.096	0.878	1.196	5.597	0.277	6.855	4.210	4.998
30	0.796	0.587	0.876	0.772	0.947	7.244	1.884	6.288	4.185	4.661
40	0.648	0.302	0.829	0.608	0.851	6.823	1.138	6.041	4.151	4.882
50	0.539	0.120	0.811	0.534	0.691	5.018	0.357	5.484	3.806	4.328

Table 6
Profit Decomposition of Equal-weighted Strategies Using Random-Portfolios

This table reports the results from implementing portfolio-based momentum strategies using all stocks on the NYSE/AMEX between 1926 and 2006. Monthly returns of individual stocks are first compounded over a six-month ranking period. We generate a uniform random number for each stock present in the ranking period, and sort stocks by the magnitude of the random number. Base portfolios of size n are then formed by sequentially picking off n stocks at a time. Specifically, n stocks with the largest random numbers are grouped in one base portfolio; the next n stocks are grouped in the next base portfolio; repeating this procedure until all stocks are assigned to portfolios. We compute equal-weighted returns for the base portfolios in the ranking period and compare them to the equal-weighted market return so as to separate winners from losers. The proceeds from selling loser portfolios are used to buy winner portfolios, with extreme winners/losers receiving the highest weights [see (11) for the weighting function]. The resulting momentum portfolio is held for the subsequent six-month holding period. The momentum profit is calculated from the equal-weighted base portfolio returns for the holding period. We maintain a one-month gap between the ranking and holding periods, in order to mitigate any market microstructure issues. This procedure is repeated in a rolling fashion for the entire sample period, and the time stamp of momentum profits range from July, 1926 to June, 2006. We allow for ten different sizes for base portfolios, that is, n ranges from 1, 2, 3, 4, 5, 10, 20, 30, 40 to 50. The four-digit numbers in the headers of the table refer to the year and the month of the stamped time, i.e., the month in the gap. For each 13-month study period, we keep track of the weighted sum of own-products of base portfolio returns, the weighted sum of cross-products of base portfolio returns, the un-weighted sum of cross-products of base portfolio returns, and the own-products of equal-weighted market returns. Note that all the own- and cross-products are defined as the product of returns in the ranking and holding periods, multiplied by 100. The time series averages of these four measures are reported below, along with heteroskedasticity and autocorrelation consistent t-statistics.

	Full Sample	Sub Sample Period				Full Sample	Sub Sample Period			
	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606
n	Panel (A) Weighted Own-products of Base Portfolios					Newey-West t-statistic				
1	1.239	2.547	0.769	0.957	0.684	3.546	2.011	3.404	2.187	2.557
2	0.620	1.274	0.384	0.478	0.342	3.546	2.011	3.404	2.187	2.557
3	0.413	0.849	0.256	0.319	0.228	3.546	2.011	3.404	2.187	2.557
4	0.310	0.637	0.192	0.239	0.171	3.546	2.011	3.404	2.187	2.557
5	0.248	0.509	0.154	0.191	0.137	3.546	2.011	3.404	2.187	2.557
10	0.124	0.255	0.077	0.096	0.068	3.546	2.011	3.404	2.187	2.557
20	0.062	0.127	0.038	0.048	0.034	3.546	2.011	3.404	2.187	2.557
30	0.041	0.085	0.026	0.032	0.023	3.546	2.011	3.404	2.187	2.557
40	0.031	0.064	0.019	0.024	0.017	3.546	2.011	3.405	2.187	2.557
50	0.025	0.051	0.015	0.019	0.014	3.546	2.011	3.404	2.187	2.557

	Full Sample	Sub Sample Period				Full Sample	Sub Sample Period			
	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606	2607-0606	2607-4606	4607-6606	6607-8606	8607-0606
n	Panel (B) Weighted Cross-products of Base Portfolios					Newey-West t-statistic				
1	0.000	0.000	0.000	0.000	0.000	-	-	-	-	-
2	0.433	1.125	0.251	0.265	0.093	2.667	1.894	2.471	1.334	0.929
3	0.560	1.483	0.331	0.342	0.084	2.629	1.908	2.450	1.308	0.644
4	0.630	1.676	0.369	0.379	0.097	2.632	1.918	2.428	1.295	0.668
5	0.655	1.744	0.391	0.391	0.094	2.600	1.901	2.424	1.256	0.605
10	0.738	1.959	0.438	0.447	0.110	2.621	1.910	2.417	1.274	0.638
20	0.780	2.075	0.464	0.469	0.112	2.618	1.912	2.427	1.271	0.617
30	0.799	2.134	0.471	0.479	0.112	2.613	1.914	2.421	1.276	0.606
40	0.804	2.146	0.475	0.482	0.113	2.610	1.910	2.427	1.273	0.603
50	0.807	2.155	0.478	0.484	0.111	2.607	1.908	2.424	1.272	0.593
n	Panel (C) Un-weighted Cross-products of Base Portfolios					Newey-West t-statistic				
1	0.000	0.000	0.000	0.000	0.000	-	-	-	-	-
2	0.867	2.250	0.502	0.530	0.186	2.667	1.894	2.471	1.334	0.929
3	0.840	2.225	0.497	0.513	0.126	2.629	1.908	2.450	1.308	0.644
4	0.840	2.235	0.492	0.506	0.129	2.632	1.918	2.428	1.295	0.668
5	0.819	2.181	0.488	0.488	0.117	2.600	1.901	2.424	1.256	0.605
10	0.821	2.177	0.486	0.496	0.122	2.621	1.910	2.417	1.274	0.638
20	0.821	2.184	0.488	0.493	0.118	2.618	1.912	2.427	1.271	0.617
30	0.826	2.207	0.487	0.496	0.116	2.613	1.914	2.421	1.276	0.606
40	0.825	2.201	0.488	0.494	0.116	2.610	1.910	2.427	1.273	0.603
50	0.824	2.199	0.488	0.494	0.114	2.607	1.908	2.424	1.272	0.593
n	Panel (D) Own-products of Market Returns					Newey-West t-statistic				
All	0.810	2.171	0.481	0.478	0.110	2.604	1.910	2.425	1.264	0.597